Contextual KP System With Minimal/ Maximal Use Of Selectors

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Abstract

In this paper, we consider contextual way of handling string objects with minimal and maximal use of selectors in KP Systems.External and Internal Contextual Grammars can not generate all the languages.One way of avoiding this draw back is to impose restrictions on the selectors used.

Keywords: Psystem, KP System, Contextual grammar , Minimal/Maximal use of selectors

1 Introduction

The research area of memberane computing originated as an attempt to formulate a model of computation motivated by the structure and functioning of

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a living cell.The direction of research was intiated in 1998 by George Paun.All classes of computing devices considered in membrane computing are called P Systems. P Systems contains a hierarchical structure of membranes placed inside a main membrane called the skin membrane. Graphically a membrane structure is represented by a venn diagram without intersection and with a unique superset. Starting from an initial configuration and using the evolution rules we get a computation.We consider a computation complete when it halts , no further rules can be applied.

A new class of P System called Kernel P System (KP System) has been introduced inorder to generalize various features like the structure of the model, the type of the rules and the execution strategy.

In this paper we define contextual KP Systems, in which the rule consists of attaching contexts to strings depending upon a choice mapping.External and internal contextual grammars canot generate all the languages.We can generate powerful languages using minimal and maximal use of selectors.

2 Prerequisites

Contextual grammars were introduced by S.Marcus as a non- chomskian model to describe natural languages. They provide an important tool in the study of formal language theory. Contextual grammars also play a major role in our understanding of grammars without the use of non terminals, called pure grammars.

In contextual grammars, the string xwy is derived by attaching the context (x,y) to the string w.

Definition 2.1. *A contextual grammar with choice is a construct.*

$$G = (V, A, C, \psi),$$

Where V *is an alphabet,* A *is a finite language over* V, C *is a finite subset of* $V^* \times V^*$ *and* ψ: $V^* \to 2^C$. *The strings in* A *are called* axioms, the elements (u,v) in C are called contexts and ψ is the choice mapping

Definition 2.2. An external contextual grammar is a contextual grammar where all derivations are based on the external mode " ex " that is, $x \Rightarrow_{ex} y$ if and only if $y = uxv$, for a context (u,v) in $\psi(x)$

Definition 2.3. An internal contextual grammar is a contextual grammar where all derivations are based on the internal mode " in " that is, $x \Rightarrow_{in} y$ if and only if $x = x_1x_2x_3,\ y = x_1ux_2vx_3$, for any $x_1, x_2, x_3, \in V^*, (u, v) \in \psi(x_2)$.

Definition 2.4. A total contextual grammar is a system

$$G = (V, A, C, \psi),$$

where V *is an alphabet,* A *is a finite language over* V, C *is a finite subset of* $V^* \times V^*$ *and* $\psi : V^* \times V^* \times V^* \to 2^c$.

Definition 2.5. For a total contextual grammar G = (V,B,C,ψ) *we define the relation* $\Rightarrow_G$ *on*V^* as follows : $x \Rightarrow_G\ y \iff\ x = x_1x_2x_3, y = x_1ux_2vx_3$ for some $x_1, x_2, x_3 \in V^*$,$(u, v) \in C$ such that $(u, v) \in \psi(x_1, x_2, x_3)$

Definition 2.6. A contextual grammar G = (V,B,C,ψ) *is said to be without choice if* $\psi(x) = C$ for all x in V*
Five basic families of languages are obtained, they are

1. *TC = the family of languages generated by total contextual grammars.*

2. *ECC = the family of languages generated by external contextual grammars.*

3. *ICC = the family of languages generated by internal contextual grammars.*

4. *EC = the family of languages generated by external contextual grammars without choice.*

5. *IC = the family of languages generated by internal contextual grammars without choice.*

Definition 2.7. Let F be a given family of languages. A contextual Grammar with F selection is a grammar $G = (V, A, (S_1, C_1), ..., (S_n, C_n))$ with $S_i \in F$, for all $1 \leq i \leq n$. We denote by $ICC(F), ECC(F)$ the families of languages generated in the internal and in the external mode, respectively, by contextual grammars with F selection.Here F will be one of the families FIN,REG,CF,CS,RE with emphasis on $F \in \{FIN, REG\}$

Definition 2.8.

Cosider a contextual grammar $G = (V, A, (S_1, C_1), ..., (S_n, C_n))$ .We define an internal derivation in the minimal local mode $x \Longrightarrow_{ml} y$ if and only if $x = x_1x_2x_3$, $y = x_1ux_2vx_3$ for $x_2 \in S_i, (u, v) \in C_i$, $1 \leq i \leq n$ and there are no x_1^1, x_2^1 , $x_3^1 \in V^*$ such that $x = x_1^1x_2^1x_3^1 \in V^*$, $x_2^1 \in S_i$ and $|x_1^1| \geq |x_1|$, $|x_3^1| \geq |x_3|$, $|x_2^1| < |x_2|$ In a minimal local mode ,we are not allowed to use as a selector a string which has atleast one proper substring which is a selector of the same production. For $\alpha \in m_l$ we define $L_\alpha(G) = \{x \in v^*/w \Longrightarrow_\alpha^* z$ for $w \in A\}$

Definition 2.9. Consider a contextual grammar
$G = (V, A, (S_1, C_1), (S_2, C_2)......., (S_n, C_n))$
we define an internal derivation in the minimal global mode $x \Longrightarrow_{mg} y$ if and only if $x = x_1\ x_2\ x_3$, $y = x_1ux_2vx_3$ for $x_2 \in S_i$, $(u, v) \in C_i$, $1 \leq i \leq n$ and there are no $x_1^1, x_2^1, x_3^1, \in V^*$ such that $x = x_1^1\ x_2^1\ x_3^1$, $x_2^1 \in S_j$ for $1 \leq j \leq n$ and $|x_1^1| \geq |x_1|$, $|x_3^1| \geq |x_3|$, $|x_2^1| < |x_2|$. In a minimal global mode ,the selectors is minimal with respect to all productions of the grammar. For $L_\alpha(G) = \{x \in v^*/w \Longrightarrow_\alpha^* z$ for $w \in A\}$

Definition 2.10. Consider a contextual grammar
$G = (V, A, (S_1, C_1), (S_2, C_2)......., (S_n, C_n))$
we define an internal derivation in the maximal local mode $x \Longrightarrow_{Ml} y$ if and only if $x = x_1x_2x_3$, $y = x_1ux_2vx_3$ for $x_2 \in S_i$, $(u, v) \in C_i$, $1 \leq i \leq n$ and there are no $x_1^1, x_2^1, x_3^1 \in v*$ such that $x = x_1^1x_2^1x_3^1$ for $x_2 \in S_i$ and $|x_1^1| \leq |x_1|$, $|x_3^1| \leq |x_3|$, $|x_2^1| > |x_2|$. In a maximal local mode ,no strict superword

of the selector can be used by the same production. For $\alpha \in M_l$ we define $L_\alpha(G) = \{x \in v^*/w \Longrightarrow_\alpha^* z \text{ for } w \in A\}$

Definition 2.11. Consider a contextual grammar
$G = (V, A, (S_1, C_1), (S_2, C_2)......., (S_n, C_n))$
we define an internal derivation in the maximal global mode $x \Longrightarrow \mathrm{M}_g\, y$ if and only if $x = x_1x_2x_3$, $y = x_1ux_2vx_3$ for $x_2 \in S_i$, $(u, v) \in C_i$, $1 \leq i \leq n$ and there are no $x_1^1, x_2^1, x_3^1 \in V^*$ such that $x = x_1^1x_2^1x_3^1$, $x_2^1 \in S_j$ for $1 \leq j \leq n$ and $|x_1^1| \leq |x_1|$, $|x_3^1| \leq |x_3|$, $|x_2^1| > |x_2|$. In a maximal global mode ,the selectors is the largest one in that position with respect to all productions of the grammar. for $\alpha \in Mg$ we define $L_\alpha(G) = \{x \in v^*/w \Longrightarrow_\alpha^* z \text{ for } w \in A\}$
The families of the languages of the form $L_\alpha(G)$ for G a contextual grammar with F choice, are denoted by $ICC_\alpha(F)$, $\alpha \in \{ml, mg, Ml, Mg\}$ here we consider $F \in \{FIN, REG\}$

3 Kernel P System (KP Systems)

A KP System is a formal model that uses some well known features of existing P System and includes some new elements and more importantly, it offers a coherent view on integrating them in to the same formalism. The system was introduced by M Gheorghe etal. Here a broad range of strategies to ues the rule against the multiset of objects available in each compartments is provided. Now will see the definition of comparments and KP System

Definition 3.1. *Given a finite set A called alphabet of elements, called objects and a finite set L , of elements called labels , a compartment is a tuple* $C = (l, \omega_0, R^\sigma)$ *where* $l \in L$ *is the label of the comparments* ω_0 *is the intial multiset over A and* R^σ *denotes the DNA code of C , which comparises the set of rules, denoted R , applied in this comparments and a regular expression* $\sigma, overLab(R)$ *the labels of the rule of R*

Definition 3.2. *A kernel P System of degree n is a tuple*
$K\Pi = (A, L, I0, \mu, C_1, C_2, ..., C_n, i_0)$

where A and L are as in definition 3.1, the alphabet and the set of labels respectively; I0 is a multisetb of objects from A, called environment; μ defines the membrane structure which is a graph (V, E) , where V are vertices, $V \subseteq L$ (the nodes are labels of these comparments) , and E edges, $C_1, C_2, ..., C_n$ are n compartments of the system - the inner part of each compartment is called region , which is delimited by a membrane , the labels of the comparments are from L and initial multiset are over A . i_0 is the output compartments where the result is obtained .

4 Contextual KP Systems

Definition *A contextual KP system is a construct*

$$K\Pi = (A, \mu, C_1, C_2, C_3..., C_n, i_0)$$

where A is a finite set of elements called objects ,
μ is a membrane structure, $C_1, C_2, ..., C_n$ are n compartments with

$$C_i = (t_i, w_i)$$

$$t_i = (R_i, \sigma_i)$$

R_i is a contextual rule of the form $(x, (u, v), tar)$ (attaching evolution rules) where $x, u, v \in A$ and $tar \in \{here, in, out\}$ and σ_i is an execution strategy in KP System, i_0 is the output compartment where the result is obtained

The family of all languages generated by contextual KP Systems of degree $n, n \geq 1$ in the mode $X \in \{IC, ICC, EC, ECC\}$ with attaching evolution rules and by using the target indications of the form $\{here, out, in\}$ *is denoted by* KCP (X, n)

5 Contextual KP Systems with Minimal / Maximal derivation

In an internal Contextual Grammar,the adjoined context depends on a substring of the derived string. Simple Contextual Grammars canot generate all the languages. By imposing restrictions on the selectors used we can generate powerful languages.

Definition :*A contextual KP System with Minimal / Maximal use of selectors is a construct* $KCP_\alpha = (A, \mu, C_1, C_2, ..., C_n, i_0)$ *Where A is called alphabet of elements* μ *is a membrane structure* $C_1, C_2, ..., C_n$ *are n compartments with*

$$C_i = (t_i, \omega_i)$$

$$t_i = (R_i, \sigma_i)$$

ω_i *is the initial multiset present in the comparment* C_i*.* R_i *is a contextual rule of the form* $(x, (u, v), tar)$ *(attaching evolution rules) where x,u,v* $\in A$ *and tar* $\in \{here, out, in\}$
x is the selector used in the minimal or maximal in the local or global mode.In the maximal global mode the entire membrane system the slector should be maximum .In maximal local mode in the particular membrane where the selector will be maximum.Similar result is valid for the minimal global mode and minimal local mode.σ_i *is an execution strategy in the KP System,*i_0 *is the output compartment where the result is obtained.*$L_\alpha(KCP)$ *denote the language generated in the contextual Kernel P System in the* α *mode where* $\alpha \in \{ml, mg, Ml, Mg\}$

Theorem 5.1. $ICC_{Ml}(REG) - ICC(REG) \neq \phi$

Proof. Consider a contextual KP System

$$KCP = (A, \mu, C_1, C_2, C_3, C_4, i_0)$$

$$A = \{a, b, c\}$$

$$\mu = [_1[_2[_3[_4]_4]_3]_2]_1$$

$$c_1 = (t_1, \omega_1)$$

$$c_2 = (t_2, \omega_2)$$

$$c_3 = (t_3, \omega_3)$$

$$c_4 = (t_4, \omega_4)$$

$$t_1 = (R_1, \sigma_1)$$

$$t_2 = (R_2, \sigma_2)$$

$$t_3 = (R_3, \sigma_3)$$

$$t_4 = (R_4, \sigma_4)$$

$$\omega_1 = \{abc\}$$

$$\omega_2 = \phi$$

$$\omega_3 = \phi$$

$$\omega_4 = \phi$$

$$i_0 = 4$$

$$R_1 = \{r_1 : (b, (a, c), in)\}$$

$$\sigma_1 = r_1$$

$$R_2 = \{r_1 : (abc, (a, c), in)\}$$

$$\sigma_2 = r_1$$

$$R_3 = \{r_1 : (a^i bc^i, (a, c), here), i \geq 2$$

$$r_2 : (a^{n-1} bc^{n-1}, (a.c), in)\}$$

$$\sigma_3 = r_1^* \, r_2$$

$$R_4 = \{r_1 : (b^i c^n, (b, \lambda), here), i \geq 1\}$$

$$\sigma_4 = r_1^*$$

Languge generated is in the maximal local mode with regular selection

$$ICC_{Ml}(REG) = \{a^n b^n c^n / n \geq 1\}$$

This language canot be generated by the internal contextual grammar with regular selection (by Marcus)

$$\{a^n b^n c^n / n \geq 1\} \notin ICC(REG)$$

Hence

$$ICC_{Ml}(REG) - ICC(REG) \neq \phi$$

□

Theorem 5.2.

$$ICC_{Ml}(REG) - ICC(REG) \neq \phi$$

Consider a contextual KP System

$$kcp = (A, \mu, c_1, i_0)$$

$$A = \{a, c\}$$

$$\mu = [_1]_1$$

$$i_0 = 1$$

$$c_1 = (R_1, \sigma_1)$$

$$t_1 = (R_1, \sigma_1)$$

$$\omega_1 = \{aca\}$$

$$R_1 = \{r_1 : (c, (a, a), here)$$

$$r_2 : (aca, (a, a), here)$$

$$r_3 : (a^2ca^2, (a, a), here)$$

....

$$r_{n-1} = (a^{n-2}ca^{n-2}, (a, a), here)\}$$

$$\sigma_1 = r_1 r_2 ... r_{n-1}$$

In the maximal local mode with regular selection, we can generate the string

$$a^n ca^n$$

$$L_{Ml}(KCP) = \{a^n ca^n / n \geq 1\}$$

This language cannot be generated in the internal contextual grammar with regular selection.

$$ICC_{Ml}(REG) - ICC(REG) \neq \phi$$

Theorem 5.3.

$$ICC_{Mg}(REG) - ICC(REG) \neq \phi$$

Consider a contextual KP System

$$KCP = (A, \mu, c_1, c_2, c_3, i_0)$$

$$A = \{a, b, c\}$$

$$\mu = [_1[_2]_2]_1$$

$$c_1 = (t_1, \omega_1)$$

$$c_2 = (t_2, \omega_2)$$

$$t_1 = (r_1, \sigma_1)$$

$$t_2 = (r_2, \sigma_2)$$

$$\omega_1 = \{bca\}$$

$$\omega_2 = \phi$$

$$i_0 = 2$$

$$R_1 = \{r_1 : (c, (b, a), here)$$

$$r_2 : (bca, (b, a), here)$$

$$.....$$

$$r_{n-1} : (b^{n-2}ca^{n-2}, (b, a), in)\}$$

$$\sigma_1 = r_1\ r_2\ ...r_{n-1}$$

$$R_2 = \{r_1 : (b^n ca^n, (a, b), here)$$

$$r_2 : (ab^n ca^n b, (a, b), here)$$

$$.........$$

$$r_n : (a^{n-1}b^n ca^n b^{n-1}, (a, b), here)\}$$

$$\sigma_2 = r_1\ r_2\ ...r_n$$

The language generated in the maximal global mode is

$$a^n b^n ca^n b^n, n \geq 1$$

This cannot be generated with ICC in the regular selection mode. Hence

$$ICC_{Mg}(REG) - ICC(REG) \neq \phi$$

Theorem 5.4.

$$ICC_{Ml}(REG) - ICC_{Mg}(FIN) \neq \phi$$

Consider a contextual KP System

$$KCP = (A, \mu, c_1, c_2, c_3, i_0)$$

$$A = \{a, b, c, d\}$$

$$\mu = [_1[_2[_3]_3]_2]_1$$

$$c_1 = (t_1, \omega_1)$$

$$c_2 = (t_2, \omega_2)$$

$$c_3 = (t_3, \omega_3)$$

$$t_1 = (r_1, \sigma_1)$$

$$t_2 = (r_2, \sigma_2)$$

$$t_3 = (r_3, \sigma_3)$$

$$\omega_1 = \{c\}$$

$$\omega_2 = \phi$$

$$\omega_3 = \phi$$

$$i_0 = 3$$

$$R_1 = \{r_1 : (c, (b, d), here)$$

$$r_2 : (bcd, (b, d), here)$$

$$r_3 : (b^2cd^2, (b, d), here)$$

$$.....$$

$$r_m : (b^{m-1}cd^{m-1}, (b, d), in)\}$$

$$\sigma_1 = r_1r_2...r_m$$

$$R_2 = \{r_1 : (b^m c, (a, \lambda), here)$$

$$r_2 : (ab^m c, (a, \lambda), here)$$

$$.....$$

$$r_n : (a^{n-1} b^m c, (a, \lambda), in)\}$$

$$\sigma_2 = r_1\ r_2 ... r_n$$

$$R_3 = \{r_1 : (b^m c, (\lambda, c), here)$$

$$r_2 : (b^m c^2, (\lambda, c), here)$$

$$.....$$

$$r_{n-1} : (b^m c^{n-1}, (\lambda, c), here)\}$$

$$\sigma_3 = r_1\ r_2 ... r_{n-1}$$

The language generated in the maximal local mode is

$$\{a^n b^m c^n d^m / n, m \geq 1$$

This language canot be generated in the maximal global mode (by Marcus) with finite selection .Hence

$$ICC_{Ml}(REG) - ICC_{Mg}(FIN) \neq \phi$$

Theorem 5.5.

$$ICC_{Ml}(REG) - ICC(REG) \neq \phi$$

Consider a contextual KP System

$$KCP = (A, \mu, c_1, c_2, i_0)$$

$$A = \{a, b, c, d\}$$

$$\mu = [_1[_2]_2]_1$$

$$c_1 = (t_1, \omega_1)$$

$$c_2 = (t_2, \omega_2)$$

$$t_1 = (r_1, \sigma_1)$$

$$t_2 = (r_2, \sigma_2)$$

$$\omega_1 = \{bc\}$$

$$\omega_2 = \phi$$

$$i_0 = 2$$

$$R_1 = \{r_1 : (bc, (a, d), here)$$

$$r_2 : (abcd, (a, d), here)$$

$$.....$$

$$r_n : (a^{n-1}bcd^{n-1}, (a, d), in)\}$$

$$\sigma_1 = r_1\ r_2...r_n$$

$$R_2 = \{r_1 : (bc, (b.c), here)$$

$$r_2 : (b^2c^2, (b, c), here)$$

$$.....$$

$$r_{n-1} : (b^{n-1}c^{n-1}, (b, c), here)\}$$

$$\sigma_2 = r_1, r_2...r_{n-1}$$

In the maximal local mode the language generated is

$$a^n b^n c^n d^n, n \geq 1$$

This language is not in the internal contextual grammar with regular choice (by Marcus) Hence

$$ICC_{Ml}(REG) - ICC(REG) \neq \phi$$

Theorem 5.6.

$$ICC_{ml}(REG) - ICC(FIN) \neq \phi$$

Consider a contextual KP System

$$KCP = (A, \mu, c_1, c_2, i_0)$$

$$A = \{a, b, \}$$

$$\mu = [_1[_2]_2]_1$$

$$c_1 = (t_1, \omega_1)$$

$$c_2 = (t_2, \omega_2)$$

$$t_1 = (r_1, \sigma_1)$$

$$t_2 = (r_2, \sigma_2)$$

$$\omega_1 = \{abab\}$$

$$\omega_2 = \phi$$

$$i_0 = 2$$

$$R_1 = \{r_1 : (ab^+a, (a, a), in)\}$$

$$\sigma_1 = r_1^*$$

$$R_2 = \{r_1 : (ba^+b, (b.b), here)\}$$

$$\sigma_2 = r_1^*$$

The language generated is

$$\{a^n b^m a^n b^m / n, m \geq 1\}$$

This language cannot be generated by the internal contextual grammer with finite selection (by Marcus). Hence

$$ICC_{ml}(REG) - ICC(FIN) \neq \phi$$

6 Conclusion

In this paper we used the contextual way of handling string objects with minimal and maximal use of selectors in KP Systems. With minimal and maximal use of selectors we can generate powerful languages that can not be generated using usual external and internal contextual grammars.

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