Duality of Iterative Soft Close in Multi Scale Soft Morphological Environment

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ABSTRACT: *In this paper, duality is discussed in soft* **1** *close in multi scale as well as iterative environment. Soft erosion and soft dilation will exist for various thresholds. So soft open and soft close also exist for various thresholds. If definition for soft erosion and soft dilation are studied (5), then some type of equalities are viewed among soft morphological operations. So equality may be established in between soft erosion and soft dilation in multi scale environment. open and close are composite operations. So soft open and soft close are also composite operations which will exist at various thresholds. Equality may be viewed among all soft morphological operations. In the same way duality also will exist for all soft morphological operations because duality will exist for all morphological operatons,for example dual of erosion is dilation ,dual of dilation is erosion,dual of open is close and dual of close is open. In this paper, duality is discussed in soft close operations , in multi scale as well as iterative environment.. A very important point is that equality does not exist in mathematical morphology but will exist in soft mathematical morphology. So various duals will occur for each soft morphological operation.*

KEY WORDS: *Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation, equality, threshold , Multi scale morphology, soft open, soft close.*

The paper is divided in to 8 sections.

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INTRODUCTION TO IMAGE PROCESSING:

IMAGE: If we observe carefully, the human beings have the desire of recording incidents, through images. Their view may be for the purpose of future generation. Images also, played the role of symbols of languages, for communication purpose.

The early cavemen documented some of the incidents through images in the caves. They documented some of the incidents of their routine life, on stones, by using primitive tools. Important incidents such as battles, routine incidents such as food habits were recorded by them, on stones. These provide record, which is historically very important, of early human civilization. The images drawn by primitive tools by Egyptians, Indians, have provided a lot of valuable information, for historians, about civilizations.

After this, paints or inks were invented. The human beings started to record scenes, incidents through these paints and inks. Letter on J. B. Porta, an Italian Philosopher, during the II half of 18th century, by mean of an accidental discovery, was able to assemble a camera like equipment by mirrors and lens, which is the first step towards the modern day photography. At the same time a France scientist observed silver chloride characteristics with respect to light. After two centuries Alexander Charles extended above concept, and produced simple photo graphs.

After one century, at around 1835 Henry Fox Talbot extended above concepts, using silver nitrate, extended the design of camera, and modern photography was born from this experiment, which is presented in royal society.

This technology is used to record incidents of U.S. civil war, or, to record incidents of wealthy people, but not reached to a common man, due to complex chemical process, for the development of photographs till "KODAK" has entered in 1884.. Later on research is done on motion pictures by Thomas A. Edison & William Kennedy Laurie Dickson, which is foundation for modern movie technology. Actually the first step for images processing was laid during Second World War. Technical experts, who are trained specially, are used to improve quality of image. They are specially trained in object recognition, they used to identity targets, manually. So, it is first step in image processing. After invention of digital computer, digital image processing came into existence. NASA, in early 1960's, got images from Space Crafts, Ranger 7, of the Lunar Surface, in thousands. These images were processed to minimize distortions. This is initial digital I.P. work, using a computer. This work was done in NASA's JET propulsion laboratory (JPL), in California.

This initial digital images processing work was very satisfactory. So, NASA continued it's funding, resulting in the development of digital image processing area.

The reduction in Hardware cost, mass production of chips, reduction in memory cost, reduction in size of computers , boosted the development of Digital Image Processing area.

So, researches in general have been showing interest and developed algorithms for image smoothening, edge enhancement, image compression, image segmentation, 2D to 3D conversion etc., Now a day, it is having applications from entertainment area to medical area.the detailed explanation is given in author's papers.

2. INTRODUCTION TO MATHEMATICAL MORPHOLOGY

At the same time mathematical morphology emerged and developed separately, with some other interests and motivations. The purpose of this area is different. But later on, it is identified that the mathematical morphology is having very important applications in image processing. So, mathematical morphology is considered now, a very important branch of image processing.

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books.

Actually the primitive operations are EROSION & DILATION. The composite operations are open and close. All these are explained in chapters 1 and 2. There are some more composite operations, like thinning, skeletenization etc. But the work is limited to erosion, dilation, open, close.

Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology (1,……. 4).

The morphological operations are suitable to apply on binary images only. Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS in this area. PETROS MARAGOS has discussed about morphology also. PETROS MARAGOS has discussed about morphology and given theoretical analysis.

For elimination or minimization of noise in the images a lot of research is done. The researchers developed algorithms for smoothening with detail preservation and for edge enhancement also. some resesrchers developed morphological algorithms for elimination of salt and pepper noise ,and impulse noise also. It has entered into medical area also

the detailed references are available in the other papers of author.(6 to 15).

3 SOFT MORPHOLOGY

The soft morphological definitions are taken from 5.

In mathematical morphology, some type of the concept "All" will play major role.In Erosion, the O.P. will be "1", if all elements of the sub image are equal to 1, otherwise, the output will be "0". In dilation, the O.P. will be "0", if all elements of the sub image are equal to "0". Otherwise the output will be "1". This "All" concept, will cause some type of inconvenience. So some type of flexibility is introduced, in the form of threshold value. So, this morphology with threshold is defined as soft morphology. So, this soft morphology is having a few advantages, which the mathematical morphology operations don't have.

So, the Soft Morphology can be considered as extension to mathematical morphology. Even though mathematical morphological operators are efficient, they suffer with a few drawbacks as specified above. In addition to above, some more comments are………. In primitive morphological operations, erosion, one or two mismatched pixels of image prevent the structuring element from fitting perfectly. It is the basic morphological operation, quantifies the way in which, the structuring element fits into the image. Erosion is an "All or nothing" transformation, implemented using bitwise "and". So, erosion will be sensitive to noise.

In primitive morphological operations, dilation, isolated pixels, even though, they are irrelevant to the image's content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise.

But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments.

So, the soft morphological filters, improve the behavior of standard morphological filters, in noisy environment. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters). It can be designed in such a way that, it performs well in removal of salt –

and – pepper noise as well as Gaussian noise, simultaneously.
The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained. The soft morphology was introduced by KOSKINEN etc, and developed by researchers.
Many researchers developed soft morphology like extending to gray tone images, statistical soft morphological filters, recursive soft morphological filters etc.and developed applications in various areas like(in addition to above mentioned applications) periodic noise reduction, elimination of disturbance, caused by solar cosmic rays, in the images obtained by astronomy base. [Solar]. Soft morphology is having advantage of designing filters in frequency domain in a very simple way. Some of the researchers go ahead one step and integrated soft morphology as well as laplacian filter and designed adaptive soft morphological laplacian filter, for smoothening as well as edge detection. adaptive soft morphological laplacian filter, for smoothening as well as edge detection.
Some more researchers designed algorithms using soft multi – scale operations for edge enhancement in noisy environment, soft segmentation ,character identification with out generating any noise. In this way even though soft morphology is very simple, its development and applications are enormous.Detailed explanation of references are given in author's papers.

4 ITERATIVE SOFT MORPHOLOGY

It can be defined as, applying a morphological operation on an image, a few number of times.
4.1 CONVENTION: symbolically, $(X \ominus Y)$ means applying erosion by S.E. Y, on image X.$(X \ominus 2Y)$ means, applying Erosion by S.E. Y, on image X, twice. $(X \ominus 3Y)$ means, applying Erosion by S.E. Y, on image X, thrice. $(X \ominus NY)$ means, applying Erosion by S.E. Y, on image "X", "N" number of times ,in the same way.
$(X \oplus NY)$ means, applying dilation by S.E. Y, on image "X" ,N no of times.(X O NY) means, applying open by S.E. Y, on image "X", N numbers if times. [But it is idempotent operation.] $(X \bullet NY)$ means applying close by S.E. Y, on image "X", N number of times. [But it is also idempotent operation.] This iterative morphology will have applications in the design of composite morphological operations (Morphological Algorithms) skeletenization, thinning, thickening etc.
The applications may also be seen in structuring element Decomposition, segmentation, etc.
Iterative morphology may be extended to iterative soft morphological environment also. In iterative soft morphological environment, the following convention may be used.

$(E_{(1)})^2$: Soft Erosion, with threshold value = 1 applied, 2 times on the image.

$(E_{(1)})^5$: Soft Erosion, with threshold value = 1 applied, 5 times on the image.

$(E_{(x)})^y$: Soft Erosion, with threshold value "x", applied "y" times on the image.

$E_{(1)}, E_{(2)}, E_{(3)}$, : Soft Erosion, applied with threshold values, 1,2,3 on the image.

$E_{(x)}, E_{(y)}, E_{(z)}$, : Soft Erosion, applied with threshold values, x,y,z on the image.

$(D_{(1)})^3$: Soft Dilation, with threshold value "1" applied "3" times on the image.

$(D_{(2)})^4$: Soft Dilation, with threshold value = 2, applied, "4" times on the image.

$(E_{(x)})^y$: Soft Dilation, with threshold value = x, applied "y" times on the image.

$D_{(1)}, D_{(2)}, D_{(3)}$: Soft Dilation, applied with threshold values, 1, 2, 3 on the image.

$D_{(x)}, D_{(y)}, D_{(z)}$: Soft Dilation, applied with threshold values x, y, z on the image.

$(O(1, 2))^3$: Soft open applied thrice on the image, with thresholds 1,2

[Soft Erosion threshold value =1, Soft Dilation threshold value =2]

$(O(x, y))^n$: Soft open, applied 'n" times, on the image, with thresholds x, y
[Soft Erosion threshold value = x, Soft Dilation threshold value = y]
O(p, q) O(x, y): Soft Open applied twice on the image, with different thresholds.
O (p, q) O (r, s) O (x, y): Soft open, applied thrice on the image, with different thresholds.
$(C(1, 2))^4$: Soft close applied four number of times on the image, with Soft dilation threshold value = 1, Soft Erosion threshold value = 2.
$[C(1, 2)]^n$: Soft close applied "n" number of times, on the image with thresholds 1, 2.
$(C(x, y))^n$: Soft close applied "n" times, on the image, with thresholds x, y.
C(p, q) C(r, s) C(t, u): Soft close applied on the image, thrice, with different thresholds.

4.2 BRIEF DISCUSSION ON ITERATIVE SOFT MORPHOLOGY:

Iterative morphology means, applying one morphological operator, on an image a few no of times. These morphological operators may have same S.E or different S.E's or same S.E with different dimensions. Iterative morphology is having its own importance. It is having so many applications in so many areas.
Iterative morphology appears in skeletonization process. In an algorithm for skeletonization erosion has to be applied, a few no of times. In thinning also, iterative morphology will appear. A Structuring

Element has to be applied so many times, on an image; [Each time the Structuring Element, will be rotated]. Same case in thickening also. Thickening also uses iterative morphological concept.

In some situations, multi scale iterative concept will appear. In multi scale skeletonization

S.E. will be applied at various dimensions, each time upon an image, to get skeletons at various dimensions.

There is an area in mathematical morphology , called S.E. decomposition. A S.E. will be divided into series of mini S.E,'s. All these S.E.'s will be applied on the image one after the other as a series or these can be applied on the image simultaneously in parallel computing environment. Any way structuring element decomposition deal with iterative morphology. The S.E. may be decomposed into mini S.E's, with dimensions in increasing order. So, S.E decomposition can be in iterative environment and multi scale environment also.

This iterative morphology can be applied for segmentation also.a few researchers proposed an algorithm for handverification also. A few researchers discussed methodology for segmentation technique, which is suitable to apply on sequence of images of traffic scene using this iterative morphology.AUPIGITER. R. also discussed segmentation using iterative watershed algorithm in 3D environment, which is suitable for medical image processing. A few researchers discussed segmentation using watershed algorithm, to be applied in medical area, using iterative erosion technique.another set of researchers discussed methodology for smoothing (for the treatment of impulse Gaussian noises) using iterative close – open technique. A researcher proposed algorithm for determination of centroids using iterative morphology. It has entered in to cluster analysis also. A few researchers discussed about applications of iterative morphology in medical image processing, regarding neurological analysis which is very important. There are so many applications, where iterative morphology may be applied. All the above works are explained in the author's other papers which are given in reference.

5 DUALITY.

Duality is a general mathematical definition which can be defined in so many ways. one way of defining is

Two operations *, . are duals to each other if $(A*B)=(A^C.B)^C$ or $(A.B)=(A^C*B)^C$

In M.M. ⊕ and ⊖ are duals. i.e. $(I \oplus S) = (I^C \ominus S)^C$ or $(I \ominus S) = (I^C \oplus S)^C$

In this discussion the following convention is followed.

If dual of "X" is "Y" then it is represented as $X^d = Y$ or $(X)^d = Y$

In Soft morphology, the duality will exist in a different ways because depending upon threshold values, many soft morphological operations will exist.

Duality will exist for each and every soft morphological operation.

Dual of E(1) is E(9) where E(1) is soft erosion at threshold 1 and E(9) is soft erosion at threshold 9.

in the same way dual of E(3) will be E(7) &. dual of D(1) will be D(9).

in the same way dual of D(1) will be D(9) & dual of D(3) will be D(7).

These concepts are explained in the author's paper 13 and 15.

In the same way multiple duals also will exist.

Multiple duals of D(1) are D(9),E(1) in 3/3 window environment.

Multiple duals of D(1) are D(25),E(1) in 5/5 window environment.

Multiple duals of D(1) are D(49),E(1) in 7/7 window environment.

Multiple duals of D(1) are D(81),E(1) in 9/9 window environment.

In the same way multiple duals of D(4) are D(6),E(4) in 3/3 window environment.

Multiple duals of D(4) are D(6),E(4) in 3/3 window environment.

Multiple duals of D(4) are D(22),E(4) in 5/5 window environment .

Multiple duals of D(4) are D(46),E(4) in 7/7 window environment.

Multiple duals of D(4) are D(78),E(4) in 9/9 window environment.

IN GENERAL

$$(D(1))^d = D(w^2), E(1)$$
$$(D(2))^d = D(w^2-1), E(2)$$
$$(D(3))^d = D(w^2-2), E(3)$$
$$(D(4))^d = D(w^2-3), E(4)$$
$$(D(5))^d = D(w^2-4), E(5)$$
$$(D(6))^d = D(w^2-5), E(6)$$
$$(D(7))^d = D(w^2-6), E(7) \quad \dots\dots\dots$$

$$\dots\dots\dots$$

$$(D(w^2-6))^d = D(7), E(w^2-6) \qquad (D(w^2-5))^d = D(6), E(w^2-5)$$
$$(D(w^2-4))^d = D(5), E(w^2-4) \qquad (D(w^2-3))^d = D(4), E(w^2-3)$$
$$(D(w^2-2))^d = D(3), E(w^2-2) \qquad (D(w^2-1))^d = D(2), E(w^2-1)$$
$$(D(w^2))^d = D(1), E(w^2)$$

In general the formula for multiple duality is

$(D(m))^d = D(w^2+1-m), E(m)$ Where $m = 1$ to w^2

The above concepts are explained in the author's paper 10.

In the same way duals of soft erosion also will exist.

The dual of soft erosion E(1) is E(9) in 3/3 window environment.
The dual of soft erosion E(1) is E(25) in 5/5 window environment.
The dual of soft erosion E(1) is E(49) in 7/7 window environment.
The dual of soft erosion E(1) is E(81) in 9/9 window environment.
The dual of soft erosion E(1) is E(121) in 11/11 window environment.
In the same way the dual of soft erosion E(5) is E(5) in 3/3 window environment.
The dual of soft erosion E(5) is E(21) in 5/5 window environment.
The dual of soft erosion E(5) is E (45) in 7/7 window environment.
The dual of soft erosion E(5) is E(77) in 9/9 window environment.
The dual of soft erosion E(5) is E(116) in 11/11 window environment.

These concepts are eloborated in paper 13.
Multiple duals of E(1) are E(9),D(1) in 3/3 window environment.
Multiple duals of E(1) are E(25),D(1) in 5/5 window environment.
Multiple duals of E(1) are E(49),D(1) in 7/7 window environment.
Multiple duals of E(1) are E(81),D(1) in 9/9 window environment.
In the same way multiple duals of E(4) are E(6),D(4) in 3/3 window environment.
Multiple duals of E(4) are E(6),D(4) in 3/3 window environment
Multiple duals of E(4) are E(22),D(4) in 5/5 window environment
Multiple duals of E(4) are E(46),D(4) in 7/7 window environment
Multiple duals of E(4) are E(78),D(4) in 9/9 window environment

IN GENERAL
$(E(1))^d = E(w^2), D(1)$ $\quad$ $(E(2))^d = E(w^2-1), D(2)$
$(E(3))^d = E(w^2-2), D(3)$ $\quad$ $(E(4))^d = E(w^2-3), D(4)$
$(E(5))^d = E(w^2-4), D(5)$ $\quad$ $(E(6))^d = E(w^2-5), D(6)$
$(E(7))^d = E(w^2-6), D(7)$

$\dots\dots\dots$
$\dots\dots\dots$

$(E(w^2-6))^d = E(7), D(w^2-6)$ $\quad$ $(E(w^2-5))^d = E(6), D(w^2-5)$
$(E(w^2-4))^d = E(5), D(w^2-4)$ $\quad$ $(E(w^2-3))^d = E(4), D(w^2-3)$
$(E(w^2-2))^d = E(3), D(w^2-2)$ $\quad$ $(E(w^2-1))^d = E(2), D(w^2-1)$
$(E(w^2))^d = E(1), D(w^2)$
In general, $(E(m))^d = E(w^2+1-m), D(m)$ Where $m = 1$ to w^2
The above concepts are explained in the author's paper 16 .

6. DUALITY OF ITERATIVE SOFT CLOSE IN MULTI SCALE SOFT MORPHOLOGICAL ENVIRONMENT

In this section the duality of iterative soft close is discussed, with various cases like same thresholds, different thresholds, and with various duals.

6.1. Duality of iterative soft close with same set of thresholds and with one type of dual.

$C(m,n) = D(m)E(n)$. (by definition)
$(C(m,n))^d = (D(m)E(n))^d = (D(m))^d(E(n))^d = E(m)D(n) = O(m,n)$
$\therefore (C(m,n))^d = O(m,n)$

$$((C(m,n))^2)^d = (C(m,n)C(m,n))^d = (C(m,n))^d(C(m,n))^d = O(m,n)O(m,n) = (O(m,n))^2$$

$$\therefore ((C(m,n))^2)^d = (O(m,n))^2$$

$$((C(m,n))^3)^d = (C(m,n)C(m,n)C(m,n))^d = (C(m,n))^d(C(m,n))^d(C(m,n))^d = O(m,n)O(m,n)O(m,n) = (O(m,n))^3$$

$$\therefore ((C(m,n))^3)^d = (O(m,n))^3$$

$$((C(m,n))^5)^d = \left(\underset{1}{C(m,n)}\underset{2}{C(m,n)}\underset{3}{C(m,n)}\dots\dots\underset{5}{C(m,n)}\right)^d = \underset{1}{(C(m,n))^d}\underset{2}{(C(m,n))^d}\underset{3}{(C(m,n))^d}\dots\dots\underset{5}{(C(m,n))^d}$$

$$= \underset{1}{O(m,n)}\underset{2}{O(m,n)}\underset{3}{O(m,n)} \dots\dots \underset{5}{O(m,n)}$$

$$= (O(m,n))^5$$

$$\therefore\ ((C(m,n))^5)^d = (O(m,n))^5$$

$$((C(m,n))^k)^d = \left(\underset{1}{C(m,n)}\underset{2}{C(m,n)}\underset{3}{C(m,n)} \dots\dots \underset{k}{C(m,n)}\right)^d = \underset{1}{(C(m,n))^d}\underset{2}{(C(m,n))^d}\underset{3}{(C(m,n))^d} \dots\dots \underset{k}{(C(m,n))^d}$$

$$= \underset{1}{O(m,n)}\underset{2}{O(m,n)}\underset{3}{O(m,n)} \dots\dots \underset{k}{O(m,n)}$$

$$= (O(m,n))^k$$

$$\therefore\ ((C(m,n))^k)^d = (O(m,n))^k$$

6.2. Duality of iterative soft close with same set of thresholds and with another type of dual.

$C(m,n) = D(m)E(n)$. (by definition)

$$\big(C(m,n)\big)^d = \big(D(m)E(n)\big)^d = \big(D(m)\big)^d\big(E(n)\big)^d = D(w^2+1-m)E(w^2+1-n)$$
$$= C(w^2+1-m, w^2+1-n)$$

$$\therefore (C(m,n))^d = C(w^2+1-m, w^2+1-n)$$

$$\left(\big(C(m,n)\big)^2\right)^d = \big(C(m,n)C(m,n)\big)^d = \big(C(m,n)\big)^d\big(C(m,n)\big)^d$$

$$= C(w^2+1-m, w^2+1-n)C(w^2+1-m, w^2+1-n)$$

$$= (C(w^2+1-m, w^2+1-n))^2$$

$$\therefore\ ((C(m,n))^2)^d = (C(w^2+1-m, w^2+1-n))^2$$

$$\left(\big(C(m,n)\big)^3\right)^d = \big(C(m,n)C(m,n)C(m,n)\big)^d = \big(C(m,n)\big)^d\big(C(m,n)\big)^d\big(C(m,n)\big)^d$$

$$= C(w^2+1-m, w^2+1-n)C(w^2+1-m, w^2+1-n)C(w^2+1-m, w^2+1-n)$$

$$= (C(w^2+1-m, w^2+1-n))^3$$

$$\therefore\ ((C(m,n))^3)^d = (C(w^2+1-m, w^2+1-n))^3$$

$$((C(m,n))^5)^d = \underset{1}{(C(m,n))^d}\underset{2}{(C(m,n))^d}\underset{3}{(C(m,n))^d} \dots\dots \underset{5}{(C(m,n))^d}$$

$$= \underset{1}{C(w^2+1-m, w^2+1-n)}\underset{2}{C(w^2+1-m, w^2+1-n)} \dots\dots \underset{5}{C(w^2+1-m, w^2+1-n)}$$

$$= \big(C(w^2+1-m, w^2+1-n)\big)^5$$

$$\therefore\ ((C(m,n))^5)^d = \big(C(w^2+1-m, w^2+1-n)\big)^5$$

$$((C(m,n))^k)^d = \underset{1}{(C(m,n))^d}\underset{2}{(C(m,n))^d}\underset{3}{(C(m,n))^d}\underset{4}{(C(m,n))^d} \dots\dots \underset{k}{(C(m,n))^d}$$

$$= \underset{1}{C(w^2+1-m, w^2+1-n)}\underset{2}{C(w^2+1-m, w^2+1-n)} \ldots\ldots \underset{k}{C(w^2+1-m, w^2+1-n)}$$

$$= \left(C(w^2+1-m, w^2+1-n)\right)^k$$

$$\therefore\ ((C(m,n))^k)^d = \left(C(w^2+1-m, w^2+1-n)\right)^k$$

6.3. Duality of iterative soft close with different sets of thresholds and with one type of dual.

$(C(a,b)C(c,d))^d(C(a,b)C(c,d))^d = (C(a,b))^d(C(c,d))^d = O(a,b)O(c,d)$

$$\therefore\ (C(a,b)C(c,d))^d = O(a,b)O(c,d)$$

$(C(a,b)C(c,d)C(e,f))^d = (C(a,b))^d(C(c,d))^d(C(e,f))^d = O(a,b)O(c,d)O(e,f)$

$$\therefore (C(a,b)C(c,d)C(e,f))^d = O(a,b)O(c,d)O(e,f)$$

$$\left(\underset{1}{C(a,b)}\underset{2}{C(c,d)}\underset{3}{C(e,f)}\underset{4}{C(g,h)} \ldots\ldots \underset{n}{C(x,y)}\right)^d = \underset{1}{(C(a,b))^d}\underset{2}{(C(c,d))^d}\underset{3}{(C(e,f))^d}\underset{4}{(C(g,h))^d} \ldots\ldots \underset{n}{(C(x,y))^d}$$

$$= \underset{1}{O(a,b)}\underset{2}{O(c,d)}\underset{3}{O(e,f)}\underset{4}{O(g,h)} \ldots\ldots \underset{n}{O(x,y)}$$

6.4. Duality of iterative soft close with different sets of thresholds and with another type of dual.

$(C(a,b))^d = C(w^2+1-a, w^2+1-b)$

$(C(a,b)C(c,d))^d = (C(a,b))^d(C(c,d))^d = C(w^2+1-a, w^2+1-b)C(w^2+1-c, w^2+1-d)$

$$\therefore\ (C(a,b)C(c,d))^d = C(w^2+1-a, w^2+1-b)C(w^2+1-c, w^2+1-d)$$

$\left(C(a,b)C(c,d)C(e,f)\right)^d = \left(C(a,b)\right)^d\left(C(c,d)\right)^d\left(C(e,f)\right)^d$
$= C(w^2+1-a, w^2+1-b)C(w^2+1-c, w^2+1-d)\, C(w^2+1-e, w^2+1-f)$

$$\therefore \left(C(a,b)C(c,d)C(e,f)\right)^d = C(w^2+1-a, w^2+1-b)C(w^2+1-c, w^2+1-d) \\ C(w^2+1-e, w^2+1-f)$$

$$\left(\underset{1}{C(a,b)}\underset{2}{C(c,d)}\underset{3}{C(e,f)}\underset{4}{C(g,h)} \ldots\ldots \underset{n}{C(x,y)}\right)^d$$

$$= \underset{1}{(C(a,b))^d}\underset{2}{(C(c,d))^d}\underset{3}{(C(e,f))^d}\underset{4}{(C(g,h))^d} \ldots\ldots \underset{n}{(C(x,y))^d}$$

$$= \underset{1}{C(w^2+1-a, w^2+1-b)}\underset{2}{C(w^2+1-c, w^2+1-d)} \ldots\ldots \underset{n}{C(w^2+1-x, w^2+1-y)}$$

6.5. Duality of two iterative soft close operations with one set of thresholds and with multiple duals.

$$(C(m,n)^2)^d = (C(m,n)C(m,n))^d = C(m,n)^d C(m,n)^d \\ = (D(m)E(n))^d\ \ (D(m)E(n))^d = (D(m))^d(E(n))^d(D(m))^d(E(n))^d$$

$$= \begin{pmatrix} \left(D(w^2+1-m), E(m)\right)\left(E(w^2+1-n), D(n)\right) \\ (D(w^2+1-m), E(m))(E(w^2+1-n), D(n)) \end{pmatrix}$$

(Put $k = w^2+1-m$ and $t = w^2+1-n$)

$$= \big(D(k), E(m)\big)\big(E(t), D(n)\big)\ \big(D(k), E(m)\big)\big(E(t), D(n)\big)$$

$$= \begin{pmatrix} \big(D(k)E(t)D(k)E(t)\big), \big(D(k)E(t)D(k)D(n)\big), \big(D(k)E(t)E(m)E(t)\big), \\ \big(D(k)E(t)E(m)D(n)\big), \big(D(k)D(n)D(k)E(t)\big), \big(D(k)D(n)D(k)D(n)\big), \\ \big(D(k)D(n)E(m)E(t)\big), \big(D(k)D(n)E(m)D(n)\big), \big(E(m)E(t)D(k)E(t)\big), \\ \big(E(m)E(t)D(k)D(n)\big), \big(E(m)E(t)E(m)E(t)\big)\big(E(m)E(t)E(m)D(n)\big), \\ (E(m)D(n)D(k)E(t)), (E(m)D(n)D(k)D(n)), (E(m)D(n)E(m)E(t)), \\ (E(m)D(n)E(m)D(n)) \end{pmatrix}$$

$$= \begin{pmatrix} \big(C(k,t)C(k,t)\big), \big(C(k,t)D(k)D(n)\big), \big(C(k,t)E(m)E(t)\big), \\ \big(C(k,t)O(m,n)\big), \big(D(k)D(n)C(k,t)\big), \big(D(k)D(n)D(k)D(n)\big), \\ \big(D(k)C(n,m)E(t)\big), \big(D(k)C(n,m)D(n)\big), \big(E(m)E(t)C(k,t)\big), \\ \big(E(m)O(t,k)D(n)\big), \big(E(m)E(t)E(m)E(t)\big), \big(E(m)E(t)O(m,n)\big), \\ \big(O(m,n)C(k,t)\big), \big(O(m,n)D(k)D(n)\big), \big(O(m,n)E(m)E(t)\big), \\ \big(O(m,n)O(m,n)\big) \end{pmatrix}$$

$$= \begin{pmatrix} \big(C(w^2+1-m, w^2+1-n)C(w^2+1-m, w^2+1-n)\big), \\ \big(C(w^2+1-m, w^2+1-n)D(w^2+1-m)D(n)\big), \\ \big(C(w^2+1-m, w^2+1-n)E(m)E(w^2+1-n)\big), \\ \big(C(w^2+1-m, w^2+1-n)O(m,n)\big), \\ \big(D(w^2+1-m)D(n)C(w^2+1-m, w^2+1-n)\big), \\ \big(D(w^2+1-m)D(n)D(w^2+1-m)D(n)\big), \\ \big(D(w^2+1-m)C(n,m)E(w^2+1-n)\big), \\ \big(D(w^2+1-m)C(n,m)D(n)\big), \\ \big(E(m)E(w^2+1-n)C(w^2+1-m, w^2+1-n)\big), \\ \big(E(m)O(w^2+1-n, w^2+1-m)D(n)\big), \\ \big(E(m)E(w^2+1-n)E(m)E(w^2+1-n)\big), \\ \big(E(m)E(w^2+1-n)O(m,n)\big), \\ \big(O(m,n)C(w^2+1-m, w^2+1-n)\big), \\ \big(O(m,n)D(w^2+1-m)D(n)\big), \\ \big(O(m,n)E(m)E(w^2+1-n)\big), \big(O(m,n)O(m,n)\big), \end{pmatrix}$$

= soft close twice or soft close and soft dilation twice or soft close and soft erosion twice or soft close and soft open or soft dilation twice and soft close or soft dilation four times or soft dilation and soft close and soft erosion or soft dilation and soft close and soft dilation or soft erosion twice and soft close or soft erosion twice and soft open and soft dilation or soft erosion four times or soft erosion twice and soft open or soft open and soft close or soft open and soft dilation twice or soft open and soft erosion twice or soft open twice.

6.6. Duality of two iterative soft close operations with two sets of thresholds and with multiple duals.

$(C(m,n)\ \ C(u,v))^d = C(m,n)^d\ \ C(u,v)^d$

$$= (D(m)E(n))^d\ \ (D(u)E(v))^d = \big(D(m)\big)^d (E(n))^d \big(D(u)\big)^d (E(v))^d$$

$$= \big(D(w^2+1-m), E(m)\big)\ \big(E(w^2+1-n), D(n)\big)$$

$$(D(w^2+1-u), E(u))\ \ (E(w^2+1-v), D(v))$$

(Put $l = w^2+1-u$ and $p = w^2+1-v$)

(Put $k = w^2+1-m$ and $t = w^2+1-n$)

$$= \big(D(k), E(m)\big)\big(E(t), D(n)\big)\,\big(D(l), E(u)\big)\big(E(p), D(v)\big)$$

$$= \begin{pmatrix} \big(D(k)E(t)D(l)E(p)\big), \big(D(k)E(t)D(l)D(v)\big), \big(D(k)E(t)E(u)E(p)\big), \\ \big(D(k)E(t)E(u)D(v)\big), \big(D(k)D(n)D(l)E(p)\big), \big(D(k)D(n)D(l)D(v)\big), \\ \big(D(k)D(n)E(u)E(p)\big), \big(D(k)D(n)E(u)D(v)\big), \big(E(m)E(t)D(l)E(p)\big), \\ \big(E(m)E(t)D(l)D(v)\big), \big(E(m)E(t)E(u)E(p)\big)\big(E(m)E(t)E(u)D(v)\big), \\ (E(m)D(n)D(l)E(p)), (E(m)D(n)D(l)D(v)), (E(m)D(n)E(u)E(p)), \\ (E(m)D(n)E(u)D(v)) \end{pmatrix}$$

$$= \begin{pmatrix} \big(C(k,t)C(l,p)\big), \big(C(k,t)D(l)D(v)\big), \big(C(k,t)E(u)E(p)\big), \\ \big(C(k,t)O(u,v)\big), \big(D(k)D(n)C(l,p)\big), \big(D(k)D(n)D(l)D(v)\big), \\ \big(D(k)C(n,u)E(p)\big), \big(D(k)C(n,u)D(v)\big), \big(E(m)E(t)C(l,p)\big), \\ \big(E(m)O(t,l)D(v)\big), \big(E(m)E(t)E(u)E(p)\big), \big(E(m)E(t)O(u,v)\big), \\ \big(O(m,n)C(l,p)\big), \big(O(m,n)D(l)D(v)\big), \big(O(m,n)E(u)E(p)\big), \\ \big(O(m,n)O(u,v)\big) \end{pmatrix}$$

$$= \begin{pmatrix} \big(C(w^2+1-m, w^2+1-n)C(w^2+1-u, w^2+1-v)\big), \\ \big(C(w^2+1-m, w^2+1-n)D(w^2+1-u)D(v)\big), \\ \big(C(w^2+1-m, w^2+1-n)E(u)E(w^2+1-v)\big), \\ \big(C(w^2+1-m, w^2+1-n)O(u,v)\big), \\ \big(D(w^2+1-m)D(n)C(w^2+1-u, w^2+1-v)\big), \\ \big(D(w^2+1-m)D(n)D(w^2+1-u)D(v)\big), \\ \big(D(w^2+1-m)C(n,u)E(w^2+1-v)\big), \\ \big(D(w^2+1-m)C(n,u)E(v)\big), \\ \big(E(m)E(w^2+1-n)C(w^2+1-u, w^2+1-v)\big), \\ \big(E(m)O(w^2+1-n, w^2+1-u)D(v)\big), \\ \big(E(m)E(w^2+1-n)E(u)E(w^2+1-v)\big), \\ \big(E(m)E(w^2+1-n)O(u,v)\big), \\ \big(O(m,n)C(w^2+1-u, w^2+1-v)\big), \\ \big(O(m,n)D(w^2+1-u)D(v)\big), \\ \big(O(m,n)E(u)E(w^2+1-v)\big), \big(O(m,n)O(u,v)\big), \end{pmatrix}$$

= soft close twice or soft close and soft dilation twice or soft close and soft erosion twice or soft close and soft open or soft dilation twice and soft close or soft dilation four times or soft dilation once and soft close once and soft erosion once or soft dilation once soft close once and soft dilation once or soft erosion twice and soft close once or soft erosion once and soft open once and soft dilation once or soft erosion four times or soft erosion twice and soft open once or soft open once and soft close once or soft open once and soft close once or soft open once and soft dilation twice or soft open once and soft erosion twice or soft open twice.

6.7. Duality of soft close with one set of thresholds and with multiple duals.

$$\begin{aligned}\big(C(m,n)\big)^d &= (D(m)\,E(n))^d = (D(m))^d\,(E(n))^d \\ &= \big(D(w^2+1-m), E(m)\big)\,\big(E(w^2+1-n), D(n)\big) \\ &= \big(D(w^2+1-m)E(w^2+1-n), E(m)E(w^2+1-n)\big), \\ &\qquad D(w^2+1-m)D(n), E(m)D(n)\end{aligned}$$

$$= \begin{pmatrix} C(w^2+1-m,\ \ w^2+1-n), E(m)E(w^2+1-n), \\ D(w^2+1-m)D(n), O(m,n) \end{pmatrix}$$

$=$ Soft close or soft erosion twice or soft dilation twice or soft open.

7. CONCLUSION: In this paper a fundamental property called DUALITY is discussed in multi scale as well as iterative environment. It will fill up gap, on the fundamentals of mathematical soft morphology. Till now applications are discussed in various papers by various researchers, but fundamental properties are not touched. More over DUALITY is having broad applications. so discussion and understanding of fundamental property in this context, will lead to development and expansion of this area, which will lead to excellent applications.

In this paper duality of soft close is discussed thouroughly and the corresponding mathematical formulae are also given which will help for further theoratical development .

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