Modeling Connectivity of Ad Hoc Network Using Fuzzy Logic & Regression Analysis

Mohit Jain
Division of Computer Engineering
Netaji Subhas Institute of Technology (NSIT)
University of Delhi
New Delhi, INDIA

Satish Chand
Division of Computer Engineering
Netaji Subhas Institute of Technology (NSIT)
University of Delhi
New Delhi, INDIA

Abstract— *One of the ad hoc network challenges is the network connectivity due to changeable and dynamic topology of network nodes. Since in ad hoc network every nodes plays the role of router, so it is possible that the movement of one or more nodes may disconnect the network. If the network get disconnected the desired data cannot be send from source to destination .So , in order to alleviate this problem we need some model that can tell us the number of connected nodes in the network for a given node density and transmission range . In this paper we propose a linear model to predict the network connectivity for the given node density and transmission range. We first use fuzzy logic to understand the connectivity by varying node density and transmission range. the resultant data is used for validation of our proposed model by using regression analysis. The R square value of our model is 0.9955, Ftest value is 178.29 and P value is 0.0001 , which indicates that our model is statistically significant .*

Keywords—*Ad hoc networks, connectivity, topology control, critical transmitting range, node density, regression , correlation ,routing ,statistical analysis system, critical points, k-connectivity, fuzzy logic.*

I. Introduction

A mobile ad hoc network is a decentralized group of mobile nodes which exchange information temporarily by means of wireless transmission. Since the nodes are mobile, the network topology may change rapidly and unpredictably over time. The network topology is unstructured and nodes may enter or leave at their will. The advantage of such networks is that the mobile devices communicate with each other in peer to peer fashion, establish a self organizing network without the need of any access point or any pre-existing infrastructure. In short, they can be formed in a spontaneous way, that’s why they are known as ad hoc networks [3-7]. One of the main properties of the MANETs is node mobility, which causes network topology to change, leading to the problem of network connectivity. A network is connected if all nodes have a communication route to each other. Maintaining connectivity is a challenge due to unstructured nature of network topology and the frequent occurrence of link and node failures due to interference, mobility, radio channel effects and battery limitations.

However, to alleviate the problem of network connectivity we develop a mathematical model which will estimate the level of connected nodes for the given node density and transmission range. Since the connectivity depends on the number of nodes per unit area and their radio transmission range, the correct adjustment of nodes radio transmission range is therefore an important system feature. In order to build the model, we first develop the fuzzy model of network connectivity by varying node density and transmission range and use the resultant data to develop the mode. To the best of author’s knowledge this is first such study that builds the mathematical model of network connectivity to know the estimate of connected nodes in the network. Our predicted equation of network connectivity will help the ad hoc researchers in building the design of ad hoc networks.

The organization of the remaining paper is as follows. Section II discusses the related literature. .Section III presents the fuzzy model followed by fuzzy simulation in section IV . Section V discuss about regression and correlation. Section VI which demonstrates the implementation of mathematical model of network connectivity .Finally the paper is concluded in section VII.

II. Literature Review

The problem of node connectivity was very first time discussed by Cheng and Robertazzi in 1989 [13]. They investigated the influence of node density and transmission range of a node‘s broadcast in a multi hop radio network modeled by spatial Poisson process. They suggested that for optimizing the transmission range, its value should be lower bounded to maintain desired network connectivity. However, they could not implement it in real scenario. The paper [14], an extension of [13], discusses the disconnectedness of Poisson distributed nodes. It provides some insights on critical coverage range vs. critical transmission range of the nodes placed in a square area according to Poisson fixed density. This problem is further discussed in [15] for one dimensional line segment that determines the critical transmitting range for Poisson distributed nodes in square area. However, these works [14, 15] are difficult to apply in real scenarios because in a Poisson process the actual number of deployed nodes is a random variable itself whose only average value can be found. The paper [16] discusses the critical power of nodes in a network for transmission to ensure network connectivity by using the percolation theory [17]. The probabilistic lower and upper bounds for isolated and connected nodes fail to explain about nodes not placed independently in a disc. The paper [18] investigates the connectivity of hybrid ad hoc networks consisting the Poisson distributed nodes using the percolation theory [17]. It reports that for populated regions one dim sparse network is well suited and the pure ad hoc network is useful relatively low density areas. For density critical areas the cellular network can provide the acceptable connectivity. The paper [19] estimates the nearest neighbors when network becomes disconnected. In this work, the same number of nearest neighbors is maintained for each node. If each node is connected to less than 0.074logn nearest neighbors the network is asymptotically disconnected. If each node is connected to more than 5.1774logn nearest neighbors, then it is asymptotically connected. The paper [20] discusses the connectivity augmentation problem and determines a set of edges of minimum weight to be inserted in order to make the resulting graph λ-vertex edge connected. It is reported that the problem is NP hard for λ>1. In [21], the same work has been discussed by using the concept of minimum geometric disk cover (MGDC) problem, commonly used in wireless networking applications or facility location problems. The paper [22] uses the random graph theory and theory of kolmogorov complexity to establish the network connectivity via building local cluster head connections between nearby cluster heads without considering global network topology. In [23], the radio transmission range problem is analyzed and the probabilistic bounds for isolated nodes and connected nodes with uniform nodes on 1- 2- and 3-dimension are calculated. It reports that the transmitting range of nodes can be reduced substantially from the deterministic requirements if there is high probability of connectedness. The paper [24] extends the work [23] and discusses the asymptotic minimum node degree of a graph on uniform points in d dimension .Some more studies on radio transmission range problem are discussed in [25-30]. The works [25-28] however do not consider inhomogeneous nodes .The issue of k-connectivity with respect to different transmission ranges has been discussed in [26, 27]. The same has been analyzed using the stochastic connectivity properties of the wireless multi hop networks in [29]. The paper [30] discusses the connectivity for inhomogeneous node distributions with random waypoint (RWP) nodes. The paper [31] extends the Bettsetter's work [30] by incorporating the deployment border effects on the range to provide k-connectivity. In [32], the critical transmission power based on Bettsetter [30] is discussed to maintain k-connectivity, ensuring k-neighbors of a node is a necessary condition but not the sufficient condition for k-connectivity. It is because the network graph may have critical points which can cause the network failure and destroys the end-to-end network connectivity. In [33, 34], the critical transmitting range for connectivity in both stationary and mobile ad hoc networks has been analyzed. The paper [33] also discusses the probability for establishing a multi hop path between two Poisson distributed nodes on an infinite line with a given distance. The paper [35] discusses about the node that keeps a multi hop path to a fixed base station with the nodes moving in a straight line away from the base station. The k-connectivity concept has been further extended in [36] that studies the critical number of neighbors needed for k-connectivity. There are critical or weak points that play a major role in destroying the network connectivity. The works [26, 27, 29, 31 36] have not discussed critical points.The paper [37] characterizes the critical transmitting range by using the asymptotic distribution of the longest minimum spanning tree [38, 39]. In [40], the problem of minimizing the maximum of node transmitting ranges while achieving connectedness is discussed. The basic assumption here is that the relative distance of all nodes is considered as input to the centralized topology control algorithm. In [41], a distributed topology control protocol is discussed to minimize the energy required to communicate with a given master node. In this work, every node is equipped with a GPS receiver to provide position information. Initially every node iteratively broadcasts its position to different search regions. When the node is able to calculate a set of nodes, called as its enclosure, based on the position information obtained from neighbors, this process stops. Its major drawback is that the number of iterations to determine the enclosure depends on the definition of initial search region, which affects the energy consumption of the protocol. The same problem has been analyzed in [42] using directional information obtained by using multi-directional antenna. But such setup is not possible in sensor networks because the nodes are very simple and have no centralized communication facility.

The above mentioned works have used graph theory [43], modern graph theory [44], and Probability theory [45], Statistics for spatial data [46], Random Graphs [47], Geometric Random Graphs [48] in order to work on the problem of node connectivity in MANETs. We use combined study of fuzzy logic and Regression analysis to address the network connectivity problem in ad hoc networks.

III. Fuzzy Model

To achieve a fully connected ad hoc network there must be a path between each pair of nodes. The connectivity therefore depends on the number of nodes per unit area (node density) and radio transmission range. Each single mobile node contributes to the connectivity of the entire network. Increasing the transmission power of node will increase transmission range that will help to cover more nodes via direct link. On the other hand, decreasing transmission power may lead to its isolation. It however cannot be decided to have almost a surely connected network for the given transmission ranges of the nodes. We estimate the number of nodes and transmission range needed for a given area to ensure network connectivity by fuzzy logic. We take node density and transmission range as two inputs and one output i.e. network connectivity. Both the input variables take three linguistic values: low, medium, high and the output variable has also three linguistic values: surely connected, ok connected, and poorly connected. The rule base is given by Table I

Table I. Fuzzy Rules for Network Connectivity

Node Density Transmission Range	Low	Medium	High
Low	Poorly	Ok	Surely
Medium	Ok	Surely	Surely
High	Surely	Surely	Surely

The node density parameter assumes values in the set for 500m x 500m {0.0004,0.0008,0.0012,0.0016,0.002,0.0024,0.0028,0.0032,0.0036,0.004} and for 1000m x 1000m {0.0001,0.0002,0.0003,0.0004,0.0005,0.0006,0.0007,0.0008,0.0009,0.0001} and the transmission range assumes values in the set {20,40,60,80,100,120,150,160,180,200,300} and the network connectivity assumes values in the set {10,20,30,40,50,60,70,80,90,100}.The membership function for both input parameters and output parameters are triangular as shown in Fig 2 ,Fig 3, Fig4 and Fig 5.

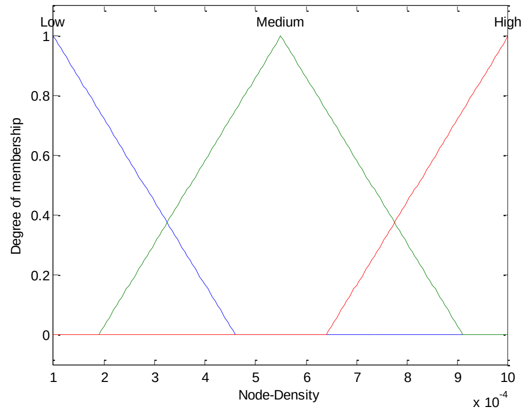


Fig 2: Illustration of Triangular Membership Function of Node Density for 500m x 500m

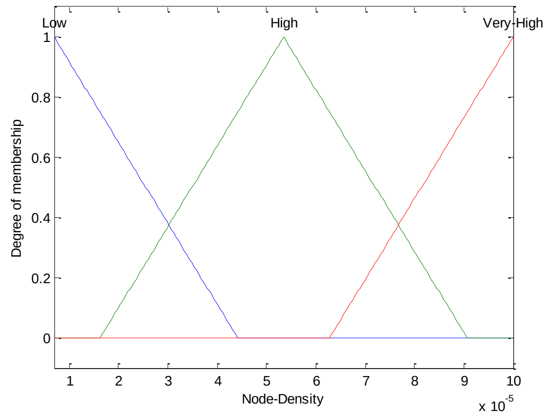


Fig 3: Illustration of Triangular Membership Function of Node Density for 1000m x 1000m

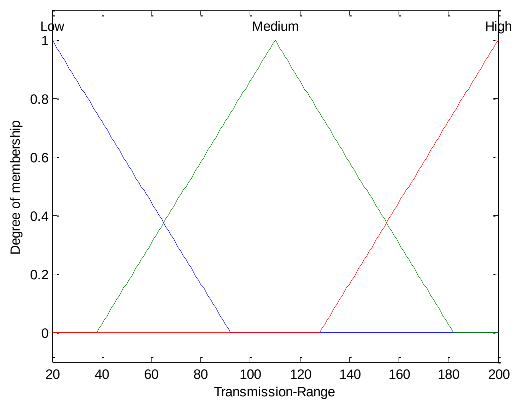


Figure 4: Illustration of Triangular Membership Function of Transmission Range

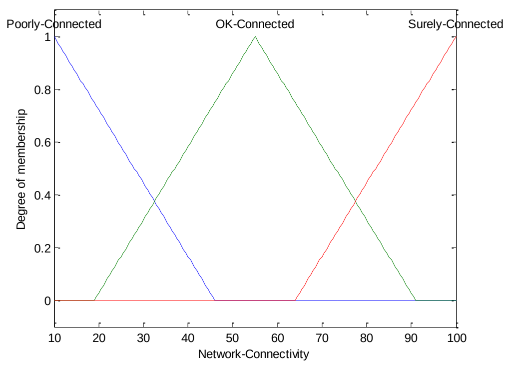


Figure 5: Illustration of Triangular Membership Curve of Network Connectivity

IV. Fuzzy Simulation

We have taken the normalized value for each parameter because the actual value of the parameters might be different for different networks. The crisp normalized values are converted into fuzzy variable. We have estimated the node density by varying the number of nodes from 7 to 100 for the given area $500x500m^2$ and $1000x1000m^2$. In fuzzy simulation [49] we have estimated the network connectivity, based on node density and transmission range, as shown in Fig6 and Fig7.

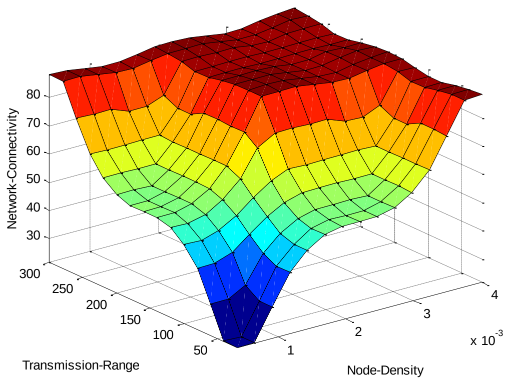


Fig6. Illustration of fuzzy simulation for terrain of size 500mx500m

Fig6 provide the information about the network connectivity which is summaries in tables II with respected to transmission range 80m, 100m, 150m and 250m for terrain of size 500m x 500m.

Table II. Analysis of Fig6 with respect to transmission range 80m, 100m, 150m, 250m

Node	Transmission Range			
Density	80	100	150	250
$0.4x10^{-3}$	38.72	53.30	80.72	81.70
$0.8x10^{-3}$	38.72	53.30	80.72	81.70
$1.2x10^{-3}$	39.70	54.21	81.56	81.92
$1.6x10^{-3}$	40.64	55.72	82.60	83.0
$2x10^{-3}$	40.64	55.74	82.60	83.12
$2.4x10^{-3}$	42.45	56.81	83.32	83.54
$2.8x10^{-3}$	42.64	57.20	83.67	83.71
$3.2x10^{-3}$	43.40	64.45	84.20	84.50
$3.6x10^{-3}$	43.56	69.20	84.67	85.0
$4x10^{-3}$	44.26	79.80	85.10	87.34

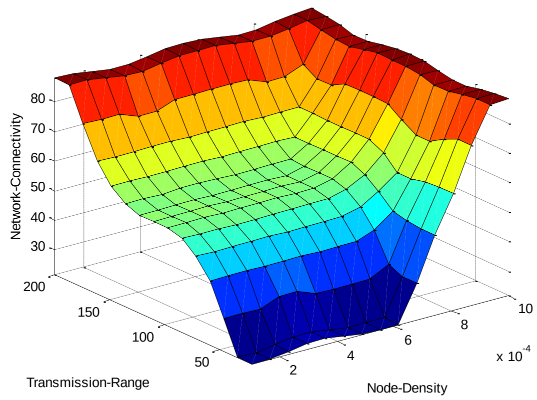


Fig7. Illustration of fuzzy simulation for terrain of size 1000m x 1000m

Fig7 provide the information about the network connectivity which is summaries in tables III with respected to transmission range 80m, 100m, 150m and 250m for terrain of size 1000m x 1000m.

Table III. Analysis of Fig7 with respect to transmission range 80m, 100m, 150m, 250m

Node	Transmission Range			
Density	80	100	150	250
$1x10^{-4}$	37.72	51.30	78.72	80.70
$2x10^{-4}$	37.78	51.32	78.74	80.72
$3x10^{-4}$	38.30	52.21	79.56	81.50
$4x10^{-4}$	38.34	52.28	79.65	81.62
$5x10^{-4}$	39.24	53.40	80.27	81.85
$6x10^{-4}$	39.30	53.48	80.35	82.12
$7x10^{-4}$	40.41	54.32	81.30	82.54
$8x10^{-4}$	40.53	54.41	81.57	82.71
$9x10^{-4}$	58.20	65.24	82.20	83.50
$10x10^{-4}$	62.56	67.20	83.67	84.54

For example, from table III we can analyze that the network connectivity is 37.72% if the node density is $1x10^{-4}$ and transmission range 80 , for 100 it will be 51.30% and so on . With the help of this data we can build mathematical model to predict the network connectivity for given node density and transmission range which has been discussed in section VI.

V. Regression and Correlation

The equation for a regression line of Y on X for the population is given as:

Y=α+βx+e –(i)

This equation is also known as the statistical model for linear regression. In this model, the expected value of Y is a linear function of X, but for fixed X , the variable Y differ from its expected value by a random amount . As a special case , Y=α+βx is called deterministic model . In this model , the actual observed value of Y is a linear function of X. In equation (i) , α is the intercept which the line cuts on the axis of Y and β is the measure of change in the dependent variable Y corresponding to a unit change in the independent variable X. If a and b are estimated values of α and β respectively, the equation of the estimated regression line is:

Y = a+bX - (ii)

Regression technique provides the actual relationship between two or more variables. It does not tell the extent of interdependence between two or more variables. In this situation, correlation methods serve our purpose. It means that change in one variable is accompanied by the proportionate change in other variable. The change is likely to occur in both the directions, i.e. the increase in one variable is accompanied by the proportionate increase in the other variable, and also the increase in one variable causes the proportionate decrease in other variable. The sample correlation is given as:

$$r_{XY} = \frac{cov(X,Y)}{\sqrt{v(X)v(Y)}}$$

Here r_{XY} is the correlation coefficient between the variables X and Y. The correlation coefficient can never be greater then 1 and less then -1. When r=1, it means that there exists a perfect positive correlation between two variables and when r=-1, it is called the perfect negative correlation. When two variables are independent, the correlation between them is zero.

To read more about regression and correlation please refer to [][].

VI. Proposed Mathematical Model

We build the mathematical model of netwok connectivity using SAS 9.2, a statistical analysis system tool . we build the model for terrain size of 1000m x 1000m for the given number of nodes with respect to transmission range of 80m ,100m ,150m and 250 m respectively . Similarly we can build the model for different terrain sizes. We have taken the data from table III in order to build the model.

a) The fuzzy dataset of network connectivity having terrain of size 1000m x 1000m with respect to transmission range 100m is given in Table IV :

Obs	node_density	network_connectivity
1	.0001	54.32
2	.0002	57.14
3	.0003	63.21
4	.0004	67.12
5	.0005	73.41
6	.0006	78.27
7	.0007	84.08
8	.0008	89.14
9	.0009	93.47
10	.0010	96.20

Table IV : Data Set

In order to build the predictive model for network connectivity having terrain of size 1000m x 1000m with respect to transmission range 100m , we consider network connectivity as dependent variable and node density as independent variable . we order the following commands in SAS system to build the model :

Proc reg data =d1;
Model network_connectivity=node_density;
Run;

The output has been displayed in fig2:

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	2020.81280	2020.81280	1782.29	<.0001
Error	8	9.07064	1.13383		
Corrected Total	9	2029.88344			

Table V: Analysis of Variance

Root MSE	1.06481	R-Square	0.9955
Dependent Mean	75.63600	Adj R-Sq	0.9950
Coeff Var	1.40781		

Table VI: Goodness of Fit

Parameter Estimates					
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	1	48.41533	0.72741	66.56	<.0001
node_density	1	49492	1172.32214	42.22	<.0001

Table VII: Parameters estimates

Interprtation : First we see that F-Test is statistically significant , which means that the model is statistically significant ,since its pvalue is less then any traditional alpha value which reject the null hypothesis that the parameters involved in regression line is statistically different from zero. The rsquare value 0.9955 means that the approximately 99% of the variance of network connectivity is accounted for by the model which is node density in this case. The t-test for node density equals 42.22 and is statistically significant which means that the regression coffecient for node density is statisticaly different from zero.

The predicted equation of network connectivity having terrain of size 1000m x 1000m with respect to transmission range 100m is :

Network_connectivity = β_0 + β_1 x node_density

where β_0 = 48.41533 and β_1 =49492. So our final equation is :

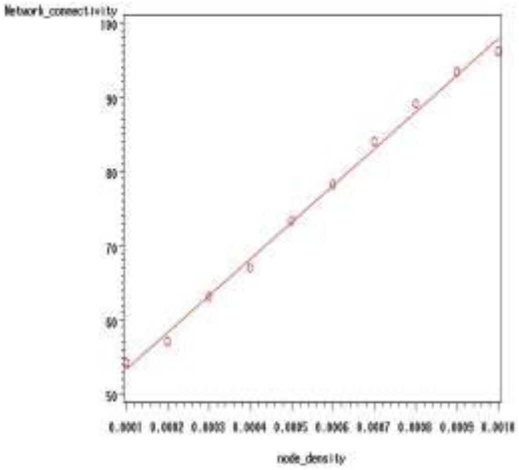
Network_connectivity = 48.41533 + 49492 x node_density .

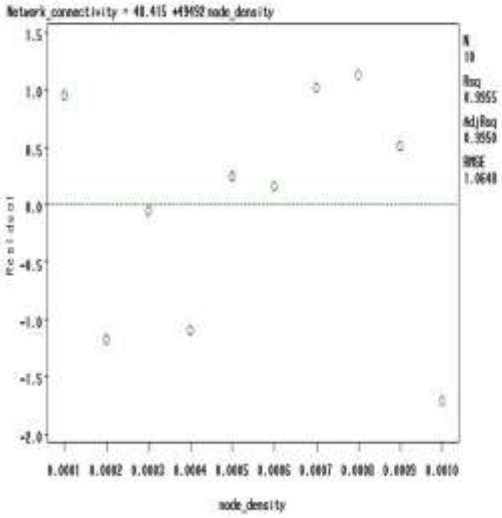
The estimated values of network connectivity has been given in the following table :

Obs	**node_density**	**network_connectivity**	**yhat**	**h**	**r**	**u**	**t**
1	.0001	54.32	53.3645	0.62585	0.95545	0.86148	1.23512
2	.0002	57.14	58.3138	0.53079	-1.17376	0.92309	1.18978
3	.0003	63.21	63.2630	0.44641	-0.05297	0.96672	1.15460
4	.0004	67.12	68.2122	0.37988	-1.09218	0.99475	1.13055
5	.0005	73.41	73.1614	0.34179	0.24861	1.00847	1.11832
6	.0006	78.27	78.1106	0.34179	0.15939	1.00847	1.11832
7	.0007	84.08	83.0598	0.37988	1.02018	0.99475	1.13055
8	.0008	89.14	88.0090	0.44641	1.13097	0.96672	1.15460
9	.0009	93.47	92.9582	0.53079	0.51176	0.92309	1.18978
10	.0010	96.20	97.9075	0.62585	-1.70745	0.86148	1.23512

Table VIII: Estimated Predicted Values of Network Connectivity

Where yhat = Predicted value of network connectivity , r= residual error , h= standard error of the indivdual predicted value , u= standard error of the mean predicted value and t= standard error of the residual . In addition to getting the regression table , it can be useful to see the regression plot between the outcome variable and predicted variable, outcome variable and residual





We can also check the interdependence of both the dependent variable and independent variable with the help of correlation . the following output explain this :

Pearson Correlation Coefficients, N = 10 Prob > |r| under H0: Rho=0		
	network_connectivity	**node_density**
network_connectivity	1.00000	0.99776 <.0001
node_density	0.99776 <.0001	1.00000

Table IX : Correlation Cofficients

Since the correlation coefficient of node density is very close to positive 1 which predict there exists a perfect positive correlation between node density and network connectivity and would be a statistically significant predictor in the regression model.

The following table summarize the mathematical model for the network connectivity having terrain of size 1000m x 1000m with respect to transmission range 80m ,100m, 150m and 250m , along with the Rsquare value which is considered as the goodness of fit for the model , which means that more the Rsqaure value better is the model .

Transmission Range	Mathematical Model ***Network_connectivity = β_0 + β_1 x node_density***	Rsquare Value
80m	***Network_connectivity = -0.00746 + 0.0020 x node_density***	0.9981
100m	***Network_connectivity = 48.41533 + 49492 x node_density***	0.9955
150m	***Network_connectivity = -0.00131 + 0.002277 x node_density***	0.9896
250m	***Network_connectivity = -0.00209 + 0.002996 x node_density***	0.9571

Table X : Mathematical model for Network Connectivity w.r.p to different transmission ranges .

VII CONCLUSION

We have proposed a mathematical model of network connectivity to estimate the level of connected nodes in the ad hoc network. With help of this model we can predict the network connectiviy for given node desnity and tranmission range before building and designing any network and can lead to high throughput and less delay of packets . The R square value of the model approximately accounts for 99% of the variance of network connectivity which shows that the model is statistically significant.

References

[1] S.S Basu and A.Chaudhari, "Self Adaptive Topology Management for Mobile Ad Hoc Network," IE(I) Journal-ET vol 84,July 2003

[2] David B.Johnson and David A.Maltz, "Dynamic Source Routing in Ad Hoc Wireless Networks,", Computer Science Department , Carneige Mellon University,5000 Forbes Avenue, Pittsburg,PA 15,213-3891.

[3] J.Jublin and J.Tornow,"The DARPA Packet Radio Network Protocols," Proc ,IEEE, vol.75, no.1,1987,pp.21-32.

[4] B.M. Leiner, D.L.Nielson, and F.A.Tobagi,eds.,Proceedings of the IEEE, Special issue on Packet Radio Networks,vol.75, Jan1987

[5] Z.J.Haas,M.Gerla,D.B.Johnson, C.E.Perkins,M.B. Pursley,M.Steenstrup, and C.-K. Toh,eds.,IEEE Journal on Sel. Areas in Comm,.Special issue on Wireless Ad Hoc Networks, vol17, August 1999.

[6] P.Kermani and N.H. Vaidya, eds.,IEEE Personal Communication, Special issue on Advances in Mobile Ad Hoc Networking, Feb 2001.

[7] C.E.Perkins,ed.,Ad Hoc Networking.Addison-Wesley,2001.

[8] L.A.Zadeh,"Fuzzy Sets",Inform. Contr., vol.8,no-3,pp 338-353,June 1965.

[9] L.X.Wang and J.M.Mendel."Generating Fuzzy Rules from Numerical Data With Applications," USC-SIPI Technique Report #169 ,1991

[10] C.C.Lee,"Fuzzy Logic in Control Systems: Fuzzy Logic Controller-Part I,"IEEE Trans. Sys.Man Cybern,vol .SMC-20,no.2,pp 408-418,Mar./April 1990.

[11] C.C.Lee,"Fuzzy Logic in Control Systems: Fuzzy Logic Controller-Part II,"IEEE Trans. Sys.Man Cybern,vol .SMC-20,no.2,pp 419-435,Mar./April 1990

[12] Network Simulator,NS-2,Homepage, http://www.isi.edu/nsnam/ns

[13] Cheng ,Y.-C.and Robertazzi,T.G. ," Critical connectivity phenomena in multihop radio models",IEEE Trans.Communication,Vol 37,1989,pp 770-777

[14] Philips,T.K.,Panwar,S.S and Tantawi,A.N. ,"Connectivity properties of a packet radio network model", IEEE Trans.Inform. Theory,Vol 35, 1989,pp 1044-1047

[15] Piret,P.," On the connectivity of radio networks", IEEE Trans.Inform.Theory,vol 37,1991,pp 1490-1492

[16] Gupta,P. and Kumar, P.R,"Critical power for asymptotic connectivity in wireless networks", In McEneany ,W.M., Optimization and Applications,1998,pp.547-566.Birkhauser

[17] R.Meesterand,R.Roy,Continuum Percolation.Cambridge,Mass.:Cambridge University Press,1996

[18] Dousse,O.,Thiran,P.and Hasler,M."Connectivity in ad hoc and hybrid networks,"In Proc.IEEEInfocom,New York,USA,June 2002, pp.1079-1088

[19] F.Xue, and P.R.Kumar,"The number of neighbors needed for connectivityof wireless networks", Wireless Networks,Vol10(2), 2004, pp.161-181

[20] S.Khuller,"Approximation algorithms for finding highly connected subgraphs",In D.S. Hochbaum, editor,Approximation algorithms for NP hard problems.PWS Publishing Co.,1996

[21] G.Ausiello,and P.Crescenzi, and G.Gambosi, and V.Kann, A.Marchetti-Spaccamela,andM.Protasi,"Complexityandapproximation:Combinatorialoptimization problems and their approximability properties",Springer-Verlag,Berlin,1999.

[22] Chlamtac,I.andFargo,A."A new approach to the design and analysis of peer to peermobilenetworks", ACM/Kluwer Wireless Networks,vol5,1999,pp.149-156

[23] Santi,P.,Blough,D.M. and Vainstein,F."A probabilistic analysis for the radio range assignmnet problem in ad hoc networks",In proc ACM mobiHoc,Long Beach,USA,October 2001,pp.212-220.

[24] M.J.B.Appel and R.P.Russo,"The minimum vertex degree of a graph on uniform points in $[0,1]^{d}$", Adv in Applied Probability,vol29,1997,pp.582-594

[25] Krishnamachari,B.,Wicker,S.B.andBejar,R.,"Phase transition phenomenon in wireless ad hoc networks",In proc.IEEE Globecom,San Antonio,USA,November 2001,pp.2921-2925

[26] Bettstetter,C.,"On the minimum node degree and connectivity of wireless multihop network",In Proc .ACM MobiHoc,June 2002,pp.80-91.

[27] Bettsetter,C.,"On the connectivty of wireless multihop networks with homogeneous and inhomogeneous range assignmnet",In Proc.IEEE VTC,Vancouver,Canada,September 2002,pp.1706-1710.

[28] Bettstetter,C. and Zangl,J.,"How to achieve a connected ad hoc network with homogeneous range assignmnet:an analytical study with considerationof border effects",In Proc.IEEE Conf.on Mobile and WirelessCommunicationNetworks(MWCN),Stockholm,Sweden,September 2002,pp.125-129

[29] Bettstetter,C.,"On the connectivity of ad hoc networks,"The Computer journal,Special issue on mobile and pervasive computing,47(4):432-447,July 2004.

[30] Bettstetter,C,"Topology properties of ad hoc networks with random waypoint mobility",In Proc.ACM MobiHoc,Annapolis,USA,June 2003,Short Paper

[31] Q.Ling and Z.Tian,"Minimum node degree and k-connectivity of a wireless multihop network in bounded area,"Proc .of IEEE Globecom,2007.

[32] H.Zang and J.C.Hou,"On the critical total power for asymptotic k-connectivity in wireless networks,"IEEE/ACM Transactions on Networking,April 2008.

[33] Santi,P.and Blough,D.M,"The critical transmitting range for connectivity in sparse wireless ad hoc networks",IEEE Trans.Mobile Computing,vol2,2003,pp.25-39.

[34] Desai,M.and Manjunath,D."On the connectivity in finite ad hoc networks",In Proc IEEE Commun.Lett.,vol6,2002,pp.437-439

[35] Nakano,K.,Shirai,Y.,Sengoku,M.and Shinoda,S."On connectivity and mobility in mobile multi hop wireless networks",In Proc.IEEE VTC,Jeju,Korea,April 2003,pp.89-98

[36] P.J.Wan and C.W.Yi,"Asymptotic critical transmission radius and critical neighbor number for k-connectivityin wireless ad hoc networks",5th ACM Symposium on Mobile Ad Hoc Networking and Computing(MobiHoc),2004.

[37] P.Panchapakesan and D.Manjunath,"On the transmission range in dense ad hoc radio networks",Proc .IEEE SPCOM'01,2001.

[38] M.D.Penrose,"A strong law for the longest edge of the minimal spanning tree,"The Annals of Probability,vol27(1),1999,pp.246-260

[39] M.D.Penrose,"The longest edge of the random minimal spanning tree,"The Annals of Probability,vol15(2),1999,pp.340-361

[40] R.Ramanathan, and R.Rosales-Hain,"Topology Control of Multihop Wireless Networks using Transmit Power Adjustment",Proc IEEE Infocom 2000,pp.404-413

[41] V.Rodolpu, and T.H.Meng,"Minimum energy mobile wireless networks",IEEE J.Selected Areas in Comm.,vol17(8),August 1999,pp.1333-1344.

[42] L.Li,J.H.Halpern,P.Bahl,Y.Wang,andR.Wattenhofer,"Analysis of a cone-based distributed topology control algorithm for

wireless multi hop networks".to appear in Proc.ACM Symp. On principles of Distributed Computing(PODC),August2001.

[43] R.Diestel,Graph theory,Springer,2nd ed..2000.

[44] B.Bollobas,Modern Graph Theory,Springer,1998.

[45] W.Feller,An Introduction to Probability theory and its Applications,John Wiley&Sons,New York,1950.

[46] S.Janson,T.Luczak,andA.Rucinski,RandomGraphs,JohnWiley&Sons,New York,2000.

[47] N.A.C.Cressie,Statistics for Spatial Data,John Wiley&Sons,1991.

[48] J.Diaz,M.D.Penrose,J.Petit,and M.Serna,"Convergence Theorems for SomeLayout Measures on Random Lattice and Random Geometric Graphs,"Combinatorics,Probability,andComputing,no.6,2000 ,pp.489-511.

[49] Maths Works ,Fuzzy Logic ToolBoox User's Guide,Jan,1998