Detection of Articulation Nodes in Mobile Ad Hoc Network Using Algebraic Graph Theory

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Abstract— *There are some points in a ad hoc network called as critical points whose failure results in partioning of the network in two or more components and makes the network disconnected and if the network become disconnected the data will not be sent to desired destination. It will lead to less throughout and delay of packets. To alleviate this problem , in this paper we proposed a new algorithm based on results from algebraic graph theory, that can find the weak points in the network for single and multiple failure cases. In addition this, the complexity of our algorithm is $O(n^2)$, which is better then previous algorithm deployed for finding critical nodes. Experimental results to evaluate the proposed algorithm to detect the nodes and links failure under network conditions are presented.*

.**Keywords—***Ad hoc networks, connectivity, topology control, critical transmitting range, node density, eigenvector, fiedler vector, Eigen values , laplacian matrix, articulation nodes*

I. Introduction

A mobile ad hoc network is a decentralized group of mobile nodes which exchange information temporarily by means of wireless transmission. Since the nodes are mobile, the network topology may change rapidly and unpredictably over time. The network topology is unstructured and nodes may enter or leave at their will. The advantage of such networks is that the mobile devices communicate with each other in peer to peer fashion, establish a self organizing network without the need of any access point or any pre-existing infrastructure. In short, they can be formed in a spontaneous way, that's why they are known as ad hoc networks [3-7]. One of the main properties of the MANETs is node mobility, which causes network topology to change, leading to the problem of network connectivity. A network is connected if all nodes have a communication route to each other. Maintaining connectivity is a challenge due to unstructured nature of network topology and the frequent occurrence of link and node failures due to interference, mobility, radio channel effects and battery limitations.

Connectivity in ad hoc network vary because of continuous movement of nodes. It is possible that movement of one or more nodes from one point to another causes network partioning since each node play the role of router in the network. The nodes which make the network portioned are known as critical nodes or articulation nodes. For example the node C in fig 1 is an articulation node that can partition the network due to its failure. Similarly the link C-D in fig2 partition the network into two components and can make the network disconnected.

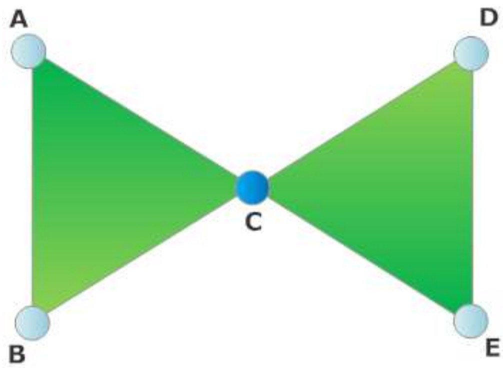

Fig 1 Critical Node C

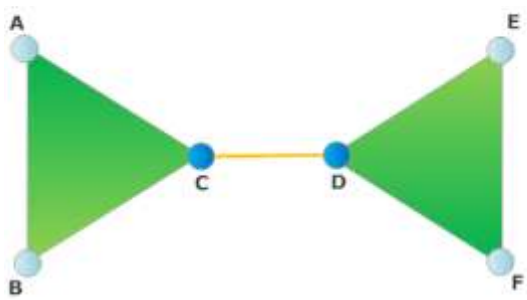

Fig 2 Critical Link C-D

To soothe this problem we have proposed the algorithm which can detect the weak points or critical points in the network that makes the network divided into two or more components.

The organization of the remaining paper is as follows. Section II discusses the related literature.

.Section III presents the articulation node identification followed by proposed algorithm. Section IV demonstrates our proposed algorithm via example. Section V illustrates the experimental results .Finally the paper is concluded in section VI.

II. Literature Review

The problem of node connectivity was very first time discussed by Cheng and Robertazzi in 1989 [13]. They investigated the influence of node density and transmission range of a node‘s broadcast in a multi hop radio network modeled by spatial Poisson process. They suggested that for optimizing the transmission range, its value should be lower bounded to maintain desired network connectivity. However, they could not implement it in real scenario. The paper [14], an extension of [13], discusses the disconnectedness of Poisson distributed nodes. It provides some insights on critical coverage range vs. critical transmission range of the nodes placed in a square area according to Poisson fixed density. This problem is further discussed in [15] for one dimensional line segment that determines the critical transmitting range for Poisson distributed nodes in square area. However, these works [14, 15] are difficult to apply in real scenarios because in a Poisson process the actual number of deployed nodes is a random variable itself whose only average value can be found. The paper [16] discusses the critical power of nodes in a network for transmission to ensure network connectivity by using the percolation theory [17]. The probabilistic lower and upper bounds for isolated and connected nodes fail to explain about nodes not placed independently in a disc. The paper [18] investigates the connectivity of hybrid ad hoc networks consisting the Poisson distributed nodes using the percolation theory [17]. It reports that for populated regions one dim sparse network is well suited and the pure ad hoc network is useful relatively low density areas. For density critical areas the cellular network can provide the acceptable connectivity. The paper [19] estimates the nearest neighbors when network becomes disconnected. In this work, the same number of nearest neighbors is maintained for each node. If each node is connected to less than 0.074logn nearest neighbors the network is asymptotically disconnected. If each node is connected to more than 5.1774logn nearest neighbors, then it is asymptotically connected. The paper [20] discusses the connectivity augmentation problem and determines a set of edges of minimum weight to be inserted in order to make the resulting graph λ-vertex edge connected. It is reported that the problem is NP hard for $\lambda>1$. In [21], the same work has been discussed by using the concept of minimum geometric disk cover (MGDC) problem, commonly used in wireless networking applications or facility location problems. The paper [22] uses the random graph theory and theory of kolmogorov complexity to establish the network connectivity via building local cluster head connections between nearby cluster heads without considering global network topology. In [23], the radio transmission range problem is analyzed and the probabilistic bounds for isolated nodes and connected nodes with uniform nodes on 1- 2- and 3-dimension are calculated. It reports that the transmitting range of nodes can be reduced substantially from the deterministic requirements if there is high probability of connectedness. The paper [24] extends the work [23] and discusses the asymptotic minimum node degree of a graph on uniform points in d dimension .Some more studies on radio transmission range problem are discussed in [25-30]. The works [25-28] however do not consider inhomogeneous nodes .The issue of k-connectivity with respect to different transmission ranges has been discussed in [26, 27]. The same has been analyzed using the stochastic connectivity properties of the wireless multi hop networks in [29]. The paper [30] discusses the connectivity for inhomogeneous node distributions with random waypoint (RWP) nodes. The paper [31] extends the Bettsetter's work [30] by incorporating the deployment border effects on the range to provide k-connectivity. In [32], the critical transmission power based on Bettsetter [30] is discussed to maintain k-connectivity, ensuring k-neighbors of a node is a necessary condition but not the sufficient condition for k-connectivity. It is because the network graph may have critical points which can cause the network failure and destroys the end-to-end network connectivity. In [33, 34], the critical transmitting range for connectivity in both stationary and mobile ad hoc networks has been analyzed. The paper [33] also discusses the probability for establishing a multi hop path between two Poisson distributed nodes on an infinite line with a given distance. The paper [35] discusses about the node that keeps a multi hop path to a fixed base station with the nodes moving in a straight line away from the base station. The k-connectivity concept has been further extended in [36] that studies the critical number of neighbors needed for k-connectivity. There are critical or weak points that play a major role in destroying the network connectivity. The works [26, 27, 29, 31 36] have not discussed critical points. The paper [37] characterizes the critical transmitting range by using the asymptotic distribution of the longest minimum spanning tree [38, 39]. In [40], the problem of minimizing the maximum of node transmitting ranges while achieving connectedness is discussed. The basic assumption here is that the relative distance of all nodes is considered as input to the centralized topology control algorithm. In [41], a distributed topology control protocol is discussed to minimize the energy required to communicate with a given master node. In this work, every node is equipped with a GPS receiver to provide position information. Initially every node iteratively broadcasts its position to different search regions. When the node is able to calculate a set of nodes, called as its enclosure, based on the position information obtained from neighbors, this process stops. Its major drawback is that the number of iterations to determine the enclosure depends on the definition of initial search region, which affects the energy consumption of the protocol. The same problem has been analyzed in [42] using directional information obtained by using multi-directional antenna. But such setup is not possible in sensor networks because the nodes are very simple and have no centralized communication facility.

The above mentioned works have used graph theory [43], modern graph theory [44], and Probability theory [45], Statistics for spatial data [46], Random Graphs [47], Geometric Random Graphs [48] in order to work on the problem of node connectivity in MANETs. In our work we make use of algebraic graph theory to find the weak points or critical points in the network.

III. Articulation Nodes Identification

Consider an arbitrary MANET topology of N nodes. Let G be the graph of the topology G = (V,E) where V is the set of nodes , { v_1 , v_2 , v_3 , ….. v_n } and E is the set of links . Table 1 summaries the notations we adopt. Let A(t) denotes the N x N adjacency matrix at time t

$$A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1N}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2N}(t) \\ . & . & & . \\ . & . & & . \\ a_{N1}(t) & a_{N2}(t) & \cdots & a_{NN}(t) \end{bmatrix}$$

where

$$a_{ij}(t) = \begin{cases} 1, & \text{if node I and j are connected} \\ 0, & \text{otherwise} \end{cases}$$

The link connectivity $a_{ij}(t)$ between two nodes depends on their transmission range and can be determined by nodes locally through the exchange of “Hello Packets”.

Let d_{ij} (t) denote the degree of node at time t. The nodal degree of matrix can be determined from the adjacency matrix A (t) by summing up the elements of the ith row or column. We define D as the diagonal matrix consisting of the degree of each node.

$$D(t) = \begin{bmatrix} d_1(t) & 0 & \cdots & 0 \\ 0 & d_2(t) & \cdots & 0 \\ 0 & 0 & \cdots & d_N(t) \end{bmatrix}$$

We assume that all the links are bidirectional. Given the adjacency matrix of the graph A(t) and nodal degree matrix of graph D(t) , we can find then laplacian matrix L(t) of graph in terms of the adjacency matrix and nodal degree matrix as

$L(t) = D(t) – A(t)$

Find the eigenvectors and Eigen values of the Laplacian matrix L (t) of graph G = (V, E) of the network topology. More formally x is an Eigen value of L (t) with corresponding eigenvector λ if it is true that $Ax = \lambda x$. An n x n matrix will have at most n of these eigenvector-Eigen value pairs, multiple eigenvectors can have the same Eigen values but any repeated eigenvector will not have different Eigen values for each repetition. Label the Eigen values so that $\lambda_1 \leq \lambda_2 \leq \ldots\ldots \leq \lambda_n$. The eigenvector x_2 which corresponds to λ_2 i.e. the second smallest Eigen value is known as the fiedler vector and is used to partition the vertices. In algebraic graph theory, it has been shown that the negative values associated with the fiedler vector are considered as poorly connected vertex and can therefore be used to partition the graph into two or more components. To know more about fiedler vector, considered the articles [44]. In our proposed algorithm we have make use of fiedler vector to estimate the articulation nodes. The basic idea behind our algorithm is that first we select the node with minimum positive value in the fiedler vector and it must be unique, then this node is critical node. After that, we look for other positive values which are higher then previous one and they should be unique in order to be critical nodes. Nodes which do not have unique values are not the critical nodes. Same process goes with negative values of nodes. First we look for node with maximum negative value and it must be unique to be critical node. Similarly, we look for other negative value nodes and they should be unique to be critical nodes. Negative nodes which are not unique are not the critical nodes.

Table 1: Notation Used

Notation	Description
G	Network Graph
V	Nodes in G
E	Links in the Network
N	Number of nodes in the network
A	Adjacency matrix of the network graph G
D	Degree matrix of the network Graph G
a_{ij}	Link between nodes i and j
L	Laplacian Matrix of Network Graph G
λ	Eigen value

<u>Proposed Algorithm for articulation point identification:</u>

Input: Given any arbitrary MANET topology of N nodes .Let G be the graph of the topology G = (V, E).
Output: Set of articulation points
Step1: Compute adjacency matrix of G.
Step2: Compute degree matrix of G.
Step3: Compute Laplacian matrix of G.

Step4: Compute Fiedler Vector and then perform following;

a) Select node in fiedler vector with minimum smallest positive value which will be unique. This node is critical node

b) Select next node in fiedler vector with positive value (Either max or min) which will be unique. This is critical node

c) The nodes which has same positive value are not the critical nodes

d) Select the node which has maximum negative value. It is the critical node.

e) Select the next node in fiedler vector with negative value (Either max or min) which will be unique. This is critical node

f) The nodes which has same negative value are not critical nodes.

IV. Illustration of Proposed Algorithm

Consider the following graph given in Figure3. In order to calculate the articulation point of the given graph, let's first find its adjacency matrix at time t

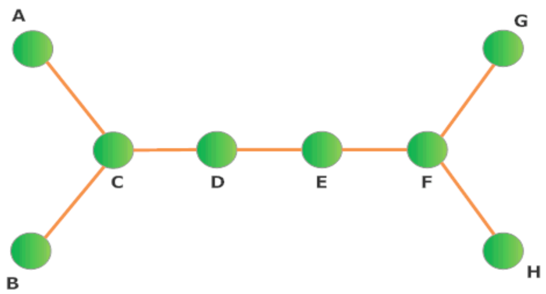

Figure 3: Illustration of graph.

A(t)=

	A	B	C	D	E	F	G	H
A	0	0	0	1	0	0	0	0
B	0	0	0	1	0	0	0	0
C	1	1	0	0	1	0	0	0
D	0	0	0	1	0	1	0	0
E	0	0	0	0	1	0	1	0
F	0	0	0	0	0	1	0	1
G	0	0	0	0	0	0	1	0
H	0	0	0	0	0	0	1	0

Degree Matrix of the given graph can be find as

D(t)=

	A	B	C	D	E	F	G	H
A	1	0	0	0	0	0	0	0
B	0	1	0	0	0	0	0	0
C	0	0	3	0	0	0	0	0
D	0	0	0	2	0	0	0	0
E	0	0	0	0	2	0	0	0
F	0	0	0	0	0	3	0	0
G	0	0	0	0	0	0	1	0
H	0	0	0	0	0	0	0	1

The Laplacian matrix L (t) for the given graph can be calculated as L (t) = D (t)-A (t). Therefore

L(t)=

	A	B	C	D	E	F	G	H
A	1	0	-1	0	0	0	0	0
B	0	1	-1	0	0	0	0	0
C	-1	-1	3	-1	0	0	0	0
D	0	0	1	2	-1	0	0	0
E	0	0	0	-1	2	-1	0	0
F	0	0	0	0	-1	3	-1	-1
G	0	0	0	0	0	-1	1	0
H	0	0	0	0	0	-1	0	1

The Eigen values and eigenvector of the Laplacian matrix L (t) is

A(t) =

	A	B	C	D	E	F	G	H
A	1	0	-1	0	0	0	0	0
B	0	0.1864	-1	0	0	0	0	0
C	-1	-1	3	-1	0	0	0	0
D	0	0	1	2	-1	0	0	0
E	0	0	0	-1	2	-1	0	0
F	0	0	0	0	-1	2.4707	-1	-1
G	0	0	0	0	0	-1	1	0
H	0	0	0	0	0	-1	0	4.3429

Here , the second smallest eigen value is 0.1864. So , the Fiedler vector corresponding to this smallest eigen value is

Field Vector =

A	-0.4267
B	-0.4267
C	-0.3472
D	-0.1234
E	0.1234
F	0.3472
G	0.4267
H	0.4267

According to our proposed algorithm, select the node which has minimum positive value from the Fiedler vector which is node E. Its value is 0.1234. So node E is our first critical nodes. Now, select the node whose value is higher then node E and it should be unique, which is node F , its value is 0.3472. so node F is our next critical node . Since the node G and Node H has the same value, so both are not the critical nodes. Now, select the node which has maximum negative value which is node D, its value is -0.1234. So node D is also the critical node. Select the next node which is lesser then node D , which is node C having the value -0.3472. So node C is also the critical node. Since node A and node B has the same negative value they are not the weak nodes.

So our final articulation nodes are: C, D, E, and F.

Similarly, we can apply this algorithm to any graph. Our algorithm is best suited for MANETs implementing proactive routing protocols where topology information is regularly gathered and it can also be used in reactive routing protocols which exchange local connectivity periodically. The time complexity of our algorithm is largely determined by computational time to determine the e\Eigen values , since it test the second smallest Eigen value to check the connectivity of the network. There are many efficient algorithm for determining Eigen values which are $O(n^2)$ where n is the size of the matrix which in our case is the number of nodes. In addition to this, our algorithm provides more information such as the number of clusters that network is portioned into and the ability to study multiple failure cases.

V. Numerical Results

In this section we illustrate the use of our proposed algorithm for detecting articulation nodes. In this study we randomly generate the network topologies with different number of nodes (20, 32,45,52,60, 74,80,92 ,100) in 1000 x 1000 m^2 using ns2 simulator , until connected topologies are found .Similarly we can generate the random network topologies with respect to any terrain size for any number of nodes. The nodes are randomly and independently distributed in the network area with (x, y) coordinates. All nodes are identical and have a 250m transmission range. The example of randomly distribution of 100 nodes in terrain of size 1000 x 1000 m^2 has been shown in figure 4. The articulation nodes test algorithm has been implemented in MATLAB and for each network topology we have compute number of single critical nodes and number of double critical nodes (i.e. Two nodes connected by an edge whose failure will result in partition of the network) based on the Fiedler vector for each network topology. Note that sparser the network more will be the articulation nodes in the network. We have plotted the node density against number of single critical nodes in figure 5 and number of double critical nodes in figure 6.

Fig Random distribution of 100 nodes in terrain size of 1000 x1000 m^2

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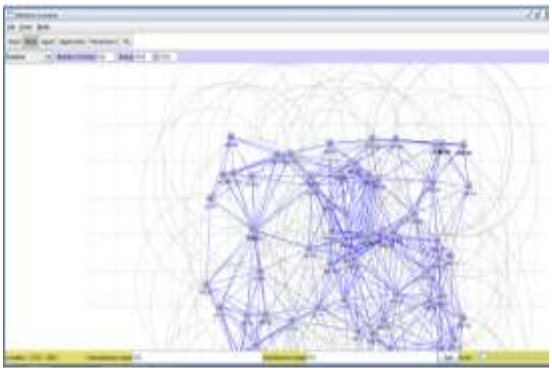

Figure 4: Random distribution of 100 nodes in terrain size of 1000m x 1000m .

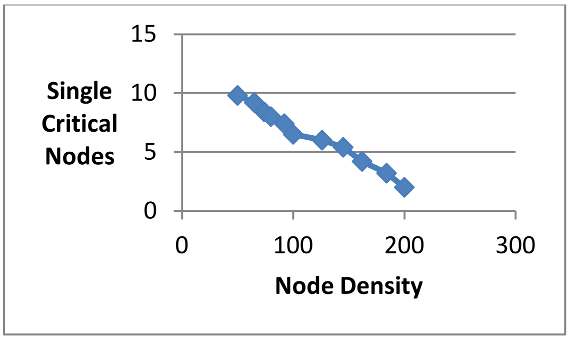

Figure5: Plot of node density with respect to single critical nodes

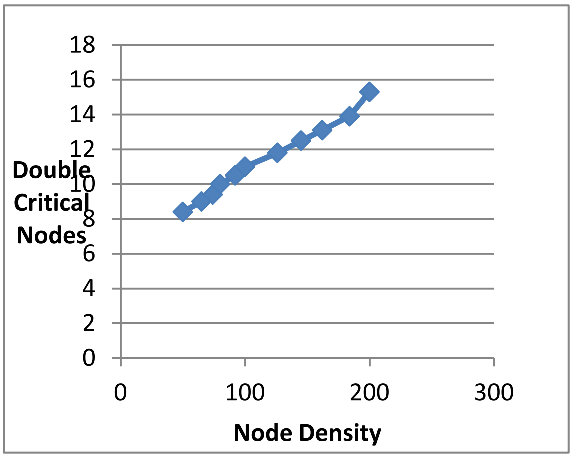

Figure 6: Plot of node density with respect to double critical nodes.

VII CONCLUSION

We have proposed a new algorithm to identify th articulation nodes in MANETs based on results from algebriac graph theory . Unlike the existing algorithms and techniques , the proposed algroithm can test for multiple failure of weak or critical points . We find that the number of single critical nodes decrease with increase in node density and the number of double critical nodes increases with increase in node density . The running time complexity of our algorithm is $O(n^2)$ which is better then previous algorithms deployed for finding the critical points. In addition to this , our algorithm provide more information such as the number of clusters that the network is partioned into and the ability to detect single and multiple failure of weak points

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