Construction of a non-linear distortion model and application to 3-D localization

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Abstract— *We present in this paper a method of modeling optical distortion for a camera with a variable focal length. This nonlinear second order model is then applied to improve 3-D calibration and localization results in monocular vision.*

Keywords— *Calibration, Distortion, focal, localization, monocular.*

I. Introduction

Concerning optical distortions, BORN and WOLF [1] assert that it is impossible to produce axially symmetrical optical systems free of all the primary geometric aberrations of SEIDEL (spherical aberration, coma, astigmatism, curvature of Field, distortion). Fortunately, only the last two affect the position and shape of the image. It is therefore important to consider these aberrations in optical models.

We will do this for the geometric distortion systematized by fig 1.

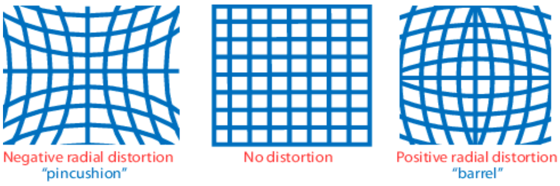


Fig. 1: distorsion

II. Selected model:

Several studies have been carried out on this subject [2], [3], [4]. The model we have chosen is inspired by that of GROSKY-TAMBURINO [5]. The solution consists in determining the nonlinear correction independently of the camera, and the model is described by the equations which follow:

$$u' = p_0 * u + p_1 * u + p_2 * u * v + p_3 * u^2 + p_4 * v^2 \quad \text{eq 1.1}$$
$$v' = q_0 * u + q_1 * u + q_2 * u * v + q_3 * u^2 + q_4 * v^2 \quad \text{eq 1.2}$$

Where u and v are the measured coordinates, u' and v' the corrected coordinates. We stopped at the order 2 because a polynomial modeling of the distortion higher than the order 2 does not improve the performance of our model. The coefficients p0, p1, p2, p3, p4, q0, q1, q2, q3 and q4 are determined by minimizing the error between the coordinates (u', v') and those measured (u, v):

$$C(u,v) = \left[(u'-u)^2 + (v'-v)^2\right]^{1/2} \quad \text{eq 2}$$

To minimize this criterion, we derive C (u, v) and then equalize to 0, ie:

$$\frac{\partial C}{\partial u} = 0 \quad \text{eq 3.1}$$
$$\frac{\partial C}{\partial v} = 0 \quad \text{eq 3.2}$$

We are thus led to solve a system of 10 non-linear equations with 10 unknowns. This criterion has been applied for different zoom positions of f1 to f5 which represent the extreme positions of the focal length.

III. Choice of points (u, v) for the development of the distortion model:

We have retained as an observed object a calibrated grid constructed with precision, the Squares are dimensional. This system makes it possible to know precisely the coordinates in pixels of the intersections of the rows and the columns which will define the retained points (u, v).

The image being much more distorted at the edges than at the center, a distribution of the points for the construction of the distortion model has been imposed in order to better take into account the characteristics of the image under consideration. In fact, we divided the image into 4 quadrants in which the points had similar and symmetrical features with respect to the center of the image. The required points were selected and were selected in a single quadrant, reducing the area of prospecting by three quarters. In practice, we chose two points near the center of the image and the other three on its edge.

IV. Results:

Once the parameters of the distortion model have been determined, we compute the coordinates u 'and v' corrected. Figs. 2.1 to 2.3 show that the optical distortion becomes more and more important as one moves away from the center of the image as well as when the focal length decreases. This distortion can reach up to 2 pixels at the periphery of the image.

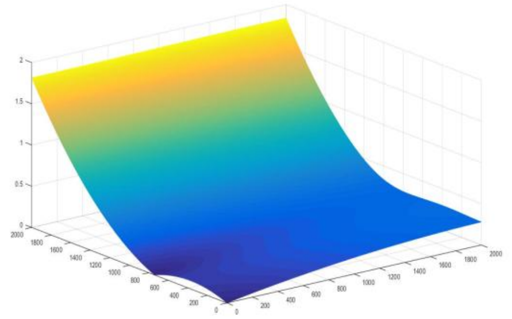


Fig 2.1

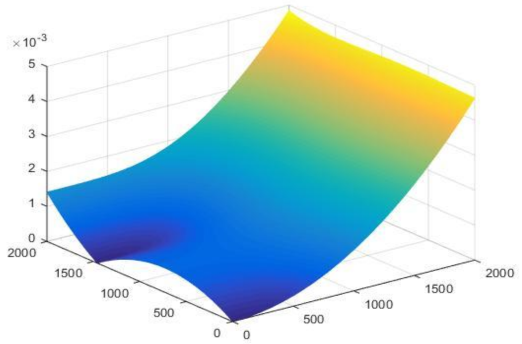


Fig 2.2

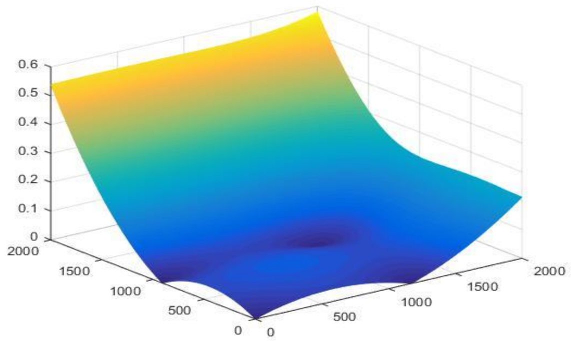


Fig 2.3

V. APPLICATION :

We have taken for application of our model of distortion the improvement of the results of the location 3-D in monocular vision

The location of polyhedral objects in computer vision requires a necessary step which is the camera calibration. This latter is to establish the relationship between the coordinates of a point in the numerated image and their 3-D homologue belonging to the reference related to the object considered. In monocular vision, many techniques in which a synthesis was made by [6] [7], use common objects in the scene: ellipses, grid ... etc .... with a significant number of points. Using a zoom, whose characteristic is obviously to determine the focal distance being tested, makes these methods outdated

The method used was developed by [8], uses common objects in the scene. It has a direct resolution and uses a minimum number of non-redundant information: 5 points. Using various transformations and changes of reference, we determine with very acceptable accuracy the position of the source and of the object in the scene.

A. Model of the camera

The retained model as shown in figure 3, is pinhole. It has the advantage of being best suited for writing equations and greatly simplifying the calculation. In this model we have:

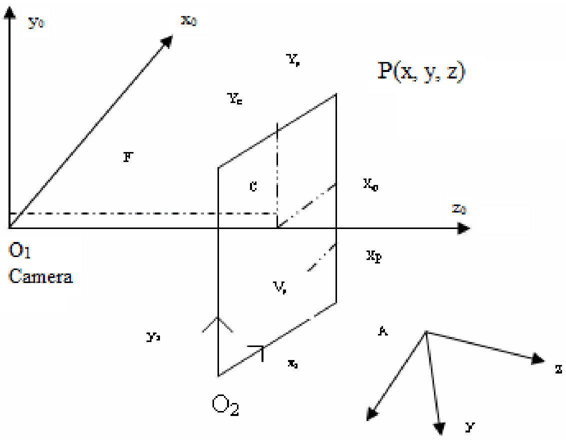


Fig 3 pinhole model

The mathematical optical axis is the line passing in O_1 and C, where C is the orthogonal projection of optical center O_1 of the image plane.

The O_1z_0 axis is carried by the mathematical optical axis.

The O_1x_0 axis is parallel to the lines of the numerated image.

(x,y,z) coordinates of the point P in the coordinate "scene" (A,x,y,z)

(X_c,Y_c): coordinates of the point C in the numerated image.

F: focal distance.

B. Position of the problem

We seek to determine the position of the focal point and the object relative to the projection plane (retina). Our unknown, defined in a reference linked to plane image are:

– Focal point position
– Object location: U, V, W origin of the object, and 3 successive rotations

It is therefore necessary to have 9 independent relationships that will give sufficient constraints to check this problem.

1) Study Subject

The object of study is a rectangular parallelepiped of known dimensions a, b and c, characterized by its rectangular face ABEC and an edge AD perpendicular to this face.
On the numerated image of the object, we carried out an operation of edge detection [9] [10] followed by a polygonal approximation to locate the coordinates with a very acceptable accuracy in the numerated image of the summits of object: (u_A, v_A) ... (u_E, v_E).

By assumption, projections of segments AB, AC, AD and AE should be distinct; any alignment result belonging to the source at a plane passing through one of the adjacent sides of the peak A and then lead to indetermination.

Transformations and changes of reference

a. Rotation γ

The first transformation given by equation (4) is to define an intermediate reference (A,x,y) whose origin is the projection (u_A,v_A) and the 2nd axis is collinear with the projection of the direction AD, which gives us:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c\gamma & s\gamma & 0 \\ -s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u-u_A \\ v-v_A \\ w-w_A \end{pmatrix}$$

equ 4

with $c\gamma = \cos\gamma$ and $s\gamma = \sin\gamma$

b. Scaling:

This 2nd transformation, applied to the coordinates of projections, allows to obtain the coordinates of the projections of the object points in a plane parallel to that of the retina and passing through the summit. Equations (2.1) to (2.5) give the coordinates of points A, B, C, D and E in the new reference:

A:(0, 0, 0) equ 5.1

B:$(wx_B, wy_B, 0)$ equ 5.2

C:$(wx_C, wy_C, 0)$ equ 5.3

D:$(0, wy_D, 0)$ equ 5.4

E:$(wx_E, wy_E, 0)$ equ 5.5

With w: strictly positive unknown designating the scaling ratio and y_D=L.

2) Reference change of the source

The source or the focal point is positioned in the retinal plane. Its position is then x_S, y_S, z_S (z_S> 0). We propose two reference rotations δ et ε which cause the source to a Ak axis of a marking (A; i, j, k) to be determined.

In the new system of axes the coordinates of the source are: S(0,0,Z)

The coordinates of the projections A, B, C, D and E become (equ (6.1) to (6.5):

A:(0,0,0)
equ 6.1

B:$(wx_B c\delta, wy_B c\varepsilon+wx_B s\delta s\varepsilon, wx_B s\delta c\varepsilon-wy_B s\varepsilon)$
equ 6.2

C:$(wx_C c\delta, wy_C c\varepsilon+wx_C s\delta s\varepsilon, wx_C s\delta c\varepsilon-wy_C s\varepsilon)$
equ 6.3

D:$(0, wy_D c\varepsilon, -wy_D s\varepsilon)$
equ 6.4

E:$(wx_E c\delta, wy_E c\varepsilon+wx_E s\delta s\varepsilon, -wy_E s\varepsilon+wx_E s\delta c\varepsilon)$
equ 6.5

with $c\delta=\cos\delta$, $s\delta=\sin\delta$, $c\varepsilon=\cos\varepsilon$ $s\varepsilon=\sin\varepsilon$

3) Positioning of the parallelepipede

The object with dimensions a, b and c, is positioned in the reference (A, i, j, k) using 2 rotations α and β, giving (equ 7.1 to 7.5):

A:(0,0,0)
eq 7.1

B:$(ac\alpha, as\alpha c\beta, as\alpha s\beta)$
eq 7.2

C:$(-bs\alpha, bc\alpha c\beta, bc\alpha s\beta)$
eq 7.3

D:$(0, -c\alpha s\beta, c\alpha c\beta)$
eq 7.4

E:$(ac\alpha-bs\alpha, as\alpha c\beta+bc\alpha c\beta, as\alpha s\beta+bc\alpha s\beta)$
eq 7.5

with $c\alpha=\cos\alpha$, $s\alpha=\sin\alpha$, $c\beta=\cos\beta$ $s\beta=\sin\beta$

4) Resolution

The above changes have reduced the size of our problem to 6 unknowns:

α and β which reflect the inclination of the object, ε and δ which reflect the inclination of the source, Z side of the source that represents the position, w: scaling ratio.

Constraints:

$\beta \in]-\pi,0[-\{-\pi/2\}$ $\alpha \in]-\pi, \pi[-\{-\pi/2,0,\pi/2\}$
$\varepsilon \in]-\pi/2, \pi/2[-\{0\}$ $\delta \in]-\pi/2, \pi/2[-\{0\}$
$Z>0$ $w>0$

The only equations that may approximate the solution are the alignment constraints between The Source S (0,0,Z), each summit of the object P (I, J, K) and its projection p (i, j, k) in the retina plane, namely: SP = λ.Sp which can also be written in projection :

$$\lambda = I / i = J / j = (K-Z) / (k-Z). \quad \text{equ 8}$$

Applying these relationships to each point B, C, D, E and their respective projections gives (equ 9.1 to 9.4):

$$B: \frac{ac\alpha}{wx_B c\gamma} = \frac{as\alpha c\beta}{wy_B c\varepsilon + wx_B s\delta s\varepsilon} = \frac{as\alpha s\beta - Z}{-wy_B s\varepsilon + wx_B s\gamma c\varepsilon - Z} \quad \text{equ 9.1}$$

$$C: \frac{-bs\alpha}{wx_C c\delta} = \frac{bc\alpha c\beta}{wy_C c\varepsilon + wx_C s\delta s\varepsilon} = \frac{bc\alpha s\beta - Z}{-wy_C s\varepsilon + wx_C s\delta c\varepsilon - Z} \quad \text{equ 9.2}$$

$$D: \frac{0}{0} = \frac{-cs\beta}{wy_D c\varepsilon} = \frac{cc\beta - Z}{-wy_D s\delta - Z} \quad \text{equ 9.3}$$

$$E: \frac{ac\alpha - bs\alpha}{wx_E c\gamma} = \frac{as\alpha c\beta + bc\alpha c\beta}{wy_E c\varepsilon + wx_E s\delta s\varepsilon} = \frac{as\alpha s\beta + bc\alpha s\beta - Z}{-wy_E s\varepsilon + wx_E s\delta c\varepsilon - Z} \quad \text{equ 9.4}$$

5) **Analytical solution**

At this stage all our unknowns are now determined analytically and we can easily calculate the camera's features and the relative position of the object.

Numerical results and discussions

In the experiment we considered a rectangular object of known size: a = 123mm, b = 115mm, c = 129mm.

Each of the faces of the object is painted with a color. On the numerated image of the object, we realize the edge detection followed by a polygonal approximation. The various data are read coordinates u and v (pixels), on the numerated image, of different object summits considered and its dimensions a, b and c.

Then for each position of the object, we will vary the focal distance between these two extremes noted f1 and f5. Table 1 shows the different parameters of the localization

Table 1 : parameters of localization

	α°	β°	ε°	δ°	w	Z (mm)
Position 1 f1 to f5	55,9 55,1 54,7 52,6 53,8	155,4 154,6 153,0 149,1 154,0	6,3 7 -7,2 -4,8 6,9	44,7 44,4 43,7 38,9 44,2	0,74 1,02 1,29 2,05 2,55	1662 1665 1668 1676 1684
Position 2 f1 to f5	63,9 64,1 59,1 40,0 62,3	130,0 128,2 127,2 134,4 124,1	15,2 14,8 -8,5 26,9 12,9	45,5 41,7 38,9 45,2 38,3	0,71 0,97 1,28 2,44 2,53	1453 1459 1464 1470 1478
Position 3 f1 to f4	76,8 74,3 74,6 74,8	113,4 113 111,6 111,9	13,4 18,2 17,7 17,6	42,6 42 38,2 39,6	1,62 1,16 0,89 0,74	1584 1581 1576 1570

We shall verify the validity of our results by comparing the value c obtained from the equation with the value c0 already known. Parameter c does not occur explicitly in the calculation

$$c = \frac{Zy_D c\varepsilon}{y_D c\beta\beta\varepsilon - s\beta\left(y_D s\varepsilon + w\right)} \quad \text{equ 10}$$

The relative error on this parameter is given by equation

$$\text{err}(c) = \left|\frac{c - c_0}{c_0}\right| \quad \text{equ 11}$$

Table 2 average error on the c parameter

	c calculated (mm)	err(c) %	Average(err(c) %)	Standard deviation
Position 1	124,7 129,2 110,2 110,5 131,0	3,3 7,5 14,5 14,3 1,2	8,2	5,4
Position 2	131,6 125,0 108,8 129,8 111,5	2 3,1 15,6 0,8 13,5	7	6,2
Position 3	140,6 124,2 120,5 120,8	8,9 3,7 6,6 6,3	6,4	1,9

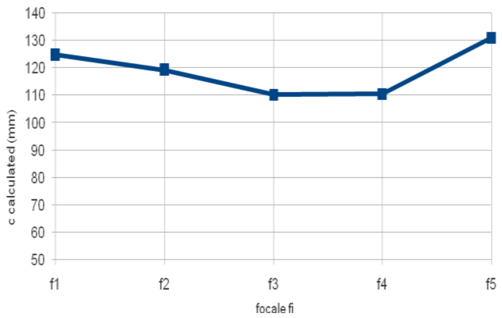
Simulation studies conducted by [8] converge to an average localization error approaching 8%, which largely corresponds to our results.

We give below (table 3) the new values of the control parameter c taking into account the model of the optical distortion calculated above.

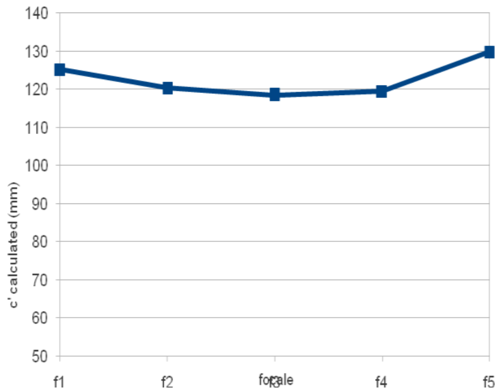
Table 3

	c' calculated (mm)	err(c') %	$err_{moy}(c')$ %
c'_0=257 mm	264,3 260,9 255,2 240,1	2,8 1,5 0,6 6,5	2,9
c'_0=139	129,2 143,6	7,5 3,3	5,4

Graphs 1 and 2 show the distribution of the parameter c calculated around the known value c = 129 mm, before and after taking into account our model of distortion. We see on graph 2 the very significant improvement of 3-D location settings.



Graph 1: before taking into account the distortion model



Graph 2: after taking into account the distortion model

Taking into account the geometric distortion, we significantly improve the localization results. Furthermore, simulation studies [3] have shown that excellent results can be obtained by this method if we get to a sub-pixel detection (0.1 to 0.2 pixel). For this, more complex distortions methods should be introduced.

VI. CONCLUSION

Our initial objective is the development of a model of optical distortion for a camera with variable focal length. The results obtained show that the distortions can reach up to 2 pixels at the periphery of the image for certain positions of the focal length. By taking this distortion model into account, we have been able to significantly improve the results of 3-D localization of polyhedral objects in monocular vision. The use of high-resolution cameras combined with sub-pixel detection methods of polyhedral object vertices as well as more complex distortion models will provide highly accurate 3-D localization results.

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