An Inventory Model for Generalized Two Parameter Weibull Distribution Deterioration and Demand Rate with Shortages

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Abstract

In this paper we have developed an inventory model for two-parameter weibull distribution deterioration and weibull demand rate with constant holding cost. The objective of this model is to minimize the total cost. Shortages are allowed during the lead time and completely backlogged. Numerical examples and graphical diagrams were used for the verification of the problem.

Keywords: *Deterioration; Shortages; Time varying holding cost; Weibull demand rate; Weibull distribution.*

1. Introduction

The recent research works in inventory control models are to maximize the profit or minimize the total cost for deteriorating item with respect to time. Deterioration arises due to some changes in the product which makes the product values dull. Deterioration in each product cannot be fully avoided and the rate of deterioration for each product will vary.

Khn-Shan wu.,[6] presented an ordering policy for items with weibull deteriorating rate and permissible delay in payments. Further, Tripathi and Pradhan [11] developed an inventory model for time propositional deterioration rate with two parameter weibull distribution and partial backlogging. Moreover Manoj Kumar Mehar, Gobinda Chandra panda and Sudhikumarsahu [7] improved an inventory model with weibull deterioration rate under the delay in payment. Recently Nandhagopal rajeswari, Thimalaisamy, and Vanjikkodi [9] have considered an inventory model for deteriorating item with two parameter weibull distribution deterioration and back logging. In continuation of the above work, Babu krishnaraj and Ramasamy [3] presented an inventory model with power demand pattern for weibull deterioration rate without shortages. Amutha and Chandrasekarn [2] have developed deteriorating inventory model for two parameter weibull demand with shortages. In addition to the above works mentioned above, Maragatham and Palani [8] have found an inventory model for deteriorating items with lead time price dependent demand and shortages.

Generally many researchers worked in stock dependent demand, price dependent demand, exponential demand, quadratic demand, power demand as demand rates. So, here we are studying weibull demand pattern for deteriorating items. Hence, we have developed an inventory model for deteriorating items with two parameter weibull demand rate. Shortages are allowed during the lead time and are completely backlogged. An analytical solution illustrated with numerical example.

2. Notations and Assumptions

A_D = Quantity of items deteriorates during a cycle time.
$\theta(t)$ = Time dependent deteriorating rate.
c = The unit cost per item
A = The ordering cost of inventory / order
$D(p)$ = Demand rate
t_1 = Replenishment cycle time
L = Lead time
P_C = Purchase cost
S_C = Shortage cost
C_H = The total cost of holding inventory per cycle.
Q = Maximum inventory level
C_D =Total deterioration cost per cycle
h = The inventory holding cost per unit item
$I(t)$ = The inventory level at time t

Assumptions:

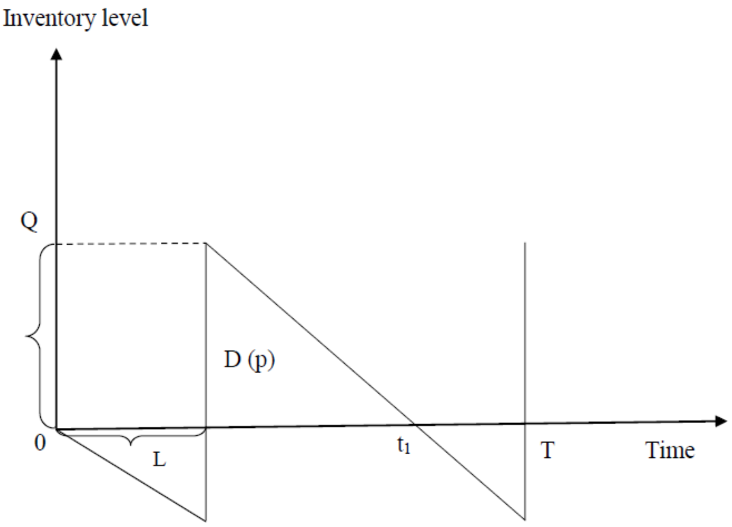
We adopt the following assumptions and notations for the model to be discussed.

1. The demand rate $D(p)$ is price depending and its of the form $D(p)=\xi\eta t^{\eta-1}$, ξ, η are demand rates.
2. The item cost remains constant irrespective of the order size.

3. Shortages are allowed.
4. Replenishment rate is infinite and the lead time is constant.
5. Holding cost is a function of time.
6. The items considered are deteriorating items but deterioration is not instantaneous.
7. The rate of deterioration at any time $t > 0$ follows the two parameter weibull distribution as $\theta = \alpha\beta t^{(\beta-1)}$, where $\alpha(0<\alpha<1)$ is the scale parameter and $\beta(>0)$ is the shape parameter.
8. There is no repair or replenishment of the deteriorated items during the inventory cycle.
9. The inventory is replenished only once in each cycle.
10. During lead time shortages are allowed.
11. Ordering quantity is $Q + LD(p)$ when $t = L$.

3. Mathematical Model and Analysis

In this model deterministic demand is weibull demand and depletion of the inventory occurs due to deterioration in each cycle. The objective of the inventory model is to determine the optimum order quantity. The holding cost is a function of time and shortages are allowed. The behaviour of inventory system at any time is shown Figure[1].



Figure[1]. Graphical Representation of the Inventory System

The rate of $I(t)$ over the cycle time T is given by the following first order differential equations.

$$\frac{d}{dt}I(t) + \alpha\beta t^{(\beta-1)}I(t) = -\xi\eta t^{(\eta-1)} \qquad L \le t \le t_1 \qquad \text{---- (1)}$$

$$\frac{d}{dt}I(t) = -\xi\eta t^{(\eta-1)} \qquad t_1 \le t \le T \qquad \text{---- (2)}$$

With the boundary conditions $I(t_1) = 0$, $I(T) = -S_1$

Solving the differential equation (1), using the boundary condition we get,

$$I(T)e^{\alpha t^{\beta}} = -\xi t^{\eta} - \alpha\xi\frac{t^{\beta+\eta}}{\beta+\eta} + C \qquad \text{---- (3)}$$

Where C is constant of integration

We obtain $I(t)$ during the time period $L \le t \le t_1$

$$I(t) = \left[\xi(t_1^{\eta} - t^{\eta}) + \alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta} - t^{\beta+\eta})\right]\left[1 - \alpha t^{\beta}\right] \quad L \le t \le t_1 \qquad \text{---- (4)}$$

$$I(t) = \xi\eta(t_1 - t) \qquad t_1 \le t \le T \qquad \text{---- (5)}$$

At time $t = L$, $I(L) = Q$, i.e., when the items are received, the level at which the organization is having a maximum inventory. So the equation (4) gives the value of Q, Where $Q + LD(p)$ is the quantity ordered at the start of the cycle.

$$Q=\left[\xi(t_1^{\eta}-L^{\eta})+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right] \qquad \text{---- (6)}$$

At $t=T$ in equation (1),

$$S_1=\xi(T^{\eta}-t_1^{\eta}),\ \text{since}\ I(t)=-S_1 \qquad \text{---- (7)}$$

The items which deteriorates during one cycle is,

$$A_D(t)=\left[\xi(t_1^{\eta}-L^{\eta})+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right]-D(t_1-L) \qquad \text{---- (8)}$$

The total variable cost will consist of the following,

(a) The ordering cost of the materials, which is fixed per order for the present financial year.

(b) The deterioration cost is given by c A_D which comes out to be

$$D_C=c\left[\xi(t_1^{\eta}-L^{\eta})+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right]-D(t_1-L) \qquad \text{---- (9)}$$

(c) The holding cost is a function of average inventory cost and it is given by ,

$$\int_L^{t_1} hI(t)dt$$

$$C_H=h\left\{\left(\xi t_1^{\eta+1}-\xi\frac{t_1^{\eta+1}}{\eta+1}-\alpha\xi\frac{t_1^{\beta+\eta+1}}{\beta+1}+\alpha\xi\frac{t_1^{\beta+\eta+1}}{\beta+\eta+1}\right)-\left(L\xi t_1^{\eta}-\xi\frac{L^{\eta+1}}{\eta+1}-\alpha\xi\frac{t_1^{\eta}L^{\beta+1}}{\beta+1}\right.\right.$$
$$\left.+\alpha\xi\frac{L^{\beta+\eta+1}}{\beta+\eta+1}\right)+\alpha\frac{\xi}{\beta+\eta}\left[\left(t_1^{\beta+\eta+1}-\frac{t_1^{\beta+\eta+1}}{\beta+\eta+1}-\alpha t_1^{2\beta+\eta+1}+\alpha\frac{t_1^{2\beta+\eta+1}}{2\beta+\eta+1}\right)\right.$$
$$\left.\left.-\left(Lt_1^{\beta+\eta}-\frac{L^{\beta+\eta+1}}{\beta+\eta+1}-\alpha t_1^{\beta+\eta}\frac{L^{\beta+1}}{\beta+1}+\alpha\frac{L^{2\beta+\eta+1}}{2\beta+\eta+1}\right)\right]\right\} \qquad \text{---- (10)}$$

(d) The purchase cost is given by

$$P_c=c[Q+LD(p)]$$
$$P_c=c\left\{\left[\xi(t_1^{\eta}-L^{\eta})+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right]+L\xi\eta t_1^{\eta-1}\right\} \qquad \text{---- (11)}$$

(e) Shortage cost is given by

$$S_C=S\left[-\int_{t_1}^{T} I(t)dt\right]$$
$$S_C=S\xi\left[\left(Tt_1^{\eta}-t_1^{\eta+1}\right)+\left(\frac{t_1^{\eta+1}-T^{\eta+1}}{\eta+1}\right)\right] \qquad \text{---- (12)}$$

Total variable cost function for one cycle is given by

$$TC=OC+P_C+H_C+D_C+S_C$$
$$TC=A+c\left\{\left[\xi(t_1^{\eta}-L^{\eta})+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right]+L\xi\eta t_1^{\eta-1}\right\}$$

$$+h\left\{\left[\left(\xi t_1^{\eta+1}-\xi\frac{t_1^{\eta+1}}{\eta+1}-\alpha\xi\frac{t_1^{\beta+\eta+1}}{\beta+1}+\alpha\xi\frac{t_1^{\beta+\eta+1}}{\beta+\eta+1}\right)-\left(L\xi t_1^{\eta}-\xi\frac{L^{\eta+1}}{\eta+1}-\alpha\xi\frac{t_1^{\eta}L^{\beta+1}}{\beta+1}\right.\right.\right.$$
$$\left.+\alpha\xi\frac{L^{\beta+\eta+1}}{\beta+\eta+1}\right)+\alpha\frac{\xi}{\beta+\eta}\left[\left(t_1^{\beta+\eta+1}-\frac{t_1^{\beta+\eta+1}}{\beta+\eta+1}-\alpha t_1^{2\beta+\eta+1}+\alpha\frac{t_1^{2\beta+\eta+1}}{2\beta+\eta+1}\right)\right.$$
$$\left.\left.\left.-\left(Lt_1^{\beta+\eta}-\frac{L^{\beta+\eta+1}}{\beta+\eta+1}-\alpha t_1^{\beta+\eta}\frac{L^{\beta+1}}{\beta+1}+\alpha\frac{L^{2\beta+\eta+1}}{2\beta+\eta+1}\right)\right]\right]\right\}+c\left\{\left[\xi(t_1^{\eta}-L^{\eta})\right.\right.$$
$$\left.\left.+\alpha\frac{\xi}{\beta+\eta}(t_1^{\beta+\eta}-L^{\beta+\eta})\right]\left[1-\alpha L^{\beta}\right]-D(t_1-L)\right\}+$$
$$S\xi\left[\left(Tt_1^{\eta}-t_1^{\eta+1}\right)+\left(\frac{t_1^{\eta+1}-T^{\eta+1}}{\eta+1}\right)\right] \qquad \text{---- (13)}$$

Our objective is to determine optimum value of Q to minimize TC. The value of t_1 for which

$\frac{\partial TC}{\partial t_1}=0$ Satisfying the condition

$$\left(\frac{\partial^2 TC}{\partial t_1^{2}}\right)>0$$

The optimal solution of the equation (13) is obtained by using Matlab. This has been illustrated by the following numerical example.

Numerical Example

We consider the following parametric values for A=300, ξ=0.02, η=0.8, c=9, α=0.005, β=0.4, L=7, h=5, S=7, D=2, T=1 year.

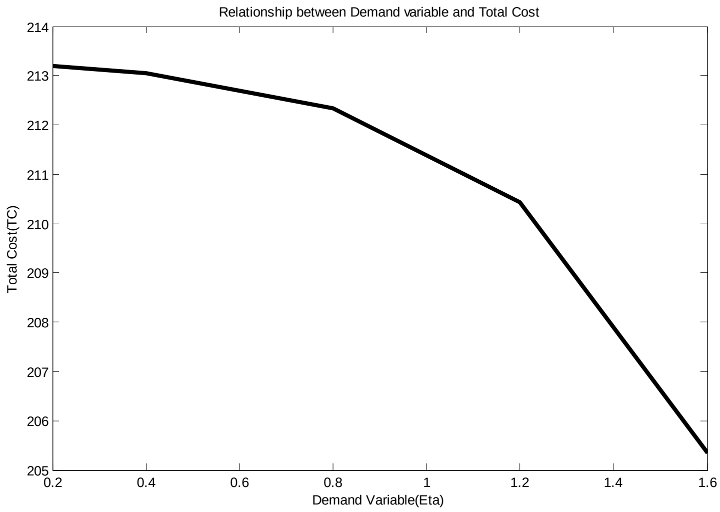
We obtain the optimal value of t_1 =11.8279, Q=0.005 and minimum Total Cost (TC) = 212.3306.

4. Sensitivity Analysis:

Table [1]

Demand Variable (η)	Maximum Inventory Level (Q)	Total Cost (TC)
0.2	0.0003	213.2005
0.4	0.0010	213.0390
0.8	0.0050	212.3306
1.2	0.0181	210.4235
1.6	0.0590	205.3459

when the demand rate(η) increases then the total cost will be decreases.



Figure[2] Relationship between Demand Variable and Total Cost

5. Conclusion

In this paper we have developed an inventory model for deteriorating items with two parameter weibull demands and two parameter weibull distribution rates with constant holding cost. This model concludes that when the demand rates (η) increases gradually the total cost decreases.

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