

Evaluating Efficacy of Forward Error Correction Coding

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Abstract—In this paper we are going to evaluate the efficacy of FEC coding. To evaluate it we are going to transfer data from Source to Destination. Before receiving the data in the Destination we create packet loss in Queue. Thus after creating packet loss, we receive the remaining packets in the Destination. Then we recover the lost Packets in the Destination and evaluate FEC's performance.

I INTRODUCTION

Packet level transport service is provided by representative packet-switched networks, including IP networks, but is not reliable and the quality-of-service (QoS) cannot be guaranteed. Packets may be lost due to buffer overflow in switching nodes, be discarded due to excessive bit errors and failure to pass the cyclic redundancy check (CRC) at the link layer, or be discarded by network control mechanisms as a response to congestion somewhere in the network.

Forward error correction (FEC) coding has often been proposed for end-to-end recovery from such packet losses. FEC can help recover the lost packets in a timely fashion through the use of redundant packets. In this paper, we will study the overall effectiveness of packet-level FEC coding, using interlaced Reed-Solomon codes, in combating network packet losses and provide an information-theoretic methodology for determining the optimum compromise between end-to-end performance and the associated increase in raw packet-loss rates using a realistic model-based analytic approach.

Reed-Solomon codes are examples of error correcting codes, in which redundant information is added to data so that it can be recovered reliably despite errors in transmission or storage and retrieval. In order to compute the weight and decode more easily, we need to use linear codes with special properties. Usually the special properties are based on algebra, in which case the code is called an algebraic code. The Reed-Solomon codes that we will now define are examples of algebraic codes. Let p be a prime number and let $m \leq n \leq p$. The Reed-Solomon code over the field Z_p with m message symbols and n code symbols is defined as follows. Given a message vector $[x_1 \ x_2 \ \dots \ x_m]$, let $P(t)$ be the polynomial

$$P(t) = x_m t^{m-1} + x_{m-1} t^{m-2} + \dots + x_2 t + x_1$$

on the other hand, from the network's perspective, the widespread use of FEC schemes by end nodes will increase the raw packet-loss rate in a network because of the additional loads resulting from transmission of redundant packets. Therefore, in order to optimize the end-to-end performance, in terms of the amount of redundancy added, and its effect on network packet-loss processes, needs to be investigated under specific and realistic modelling assumptions.

So, for a given choice of block length we expect that there is an optimum choice of redundancy, or channel coding rate, since a rate too high (low redundancy) is simply not powerful enough to effectively recover packet losses while a rate too low (high redundancy) results in excessive raw packet losses due to the increased overhead which overwhelms the packet recovery capabilities of the FEC code. The optimum channel coding rate results in an optimum compromise between these two effects.

In a packet-switched network, a flow of packets crosses a chain of routers before it reaches the destination node as shown in below figure. Most of the packet losses from a flow occur in the router which has the smallest bandwidth. Therefore, we can model the whole chain of routers in terms of this single bottleneck node. A single-multiplexer model for this bottleneck node is widely used to analyze the associated queueing-related packet losses, e.g., losses due to buffer overflows and excessive delays. Since the correlation level of the packet-loss process has great impact on the FEC efficacy, we investigate this dependence using the autocorrelation function of the packet-loss process.

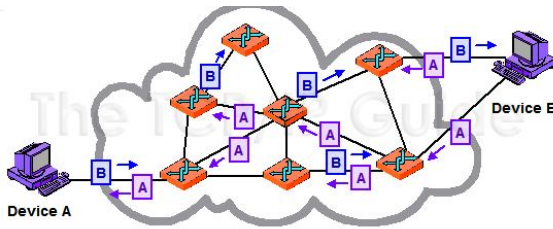


Fig.1. Packet Switching

A novel technique based on forward error correction (FEC) has been proposed that allows the destination to reconstruct missing data packets by using redundant parity packets that the source adds to each block of data packets.

In this work, we focus on evaluating the capability of FEC in recovering packet losses over IP networks using residual packet-loss rate as the performance measure. We describe the network packet transport channel in terms of a single multiplexer modeled as a G/M/1/K queue (single session) and/or an N*M/M/1/K queue (multiple sessions). This allows the use of more general packet-arrival processes and random packet lengths representative of evolving IP network applications. FEC coding performance combined with interleaving was also studied for the case of both a single session and multiple sessions. For the case of a single session, our approach to the analysis of interleaving is similar to the approach that is already described. But for the case of multiple sessions, we must provide a much simpler algorithm to analyze the effect of interleaving than the already existing approach. Using this approach we demonstrate the behavior of the resulting packet-loss statistics as a function of interleaving depth. The approach is useful in exploring the tradeoffs between coding parameters, such as interleaving depth, code rate R_c and block length. The approach is useful in exploring the tradeoffs between coding parameters, such as interleaving depth, code rate and block length.

II NETWORK MODELS USED TO EVALUATE PERFORMANCE OF FEC

Single-Multiplexer Network Model:

To compare the packet-loss statistics quantitatively here we are using one model called single-multiplexer model. As illustrated in Fig. 1, the single-multiplexer model is a queuing system which consists of three components: 1) an arrival process for packets from N different sources with corresponding packet arrival rates $\lambda_i, 1 \leq i \leq N$; 2) a buffer which can hold up to K packets, which are assumed to be served in first-come-first-served (FCFS) order; and 3) an output link with average packet service rate μ .

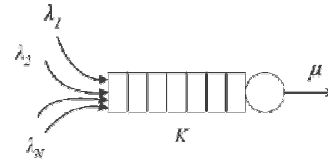


Fig. 3. Single-multiplexer network model.

The arriving packets to the multiplexer may come from a single source (N=1) or multiple- sources (N≥2). The single-source (N=1) case corresponds to a network which can apply per-flow control for the traffic, i.e., the network reserves fixed bandwidth for each traffic flow. The multiple-source (N≥2) case represents a network in which no per-flow control is applied and packets from different sources share the output bandwidth and the buffer.

III SYSTEM MODEL FOR FEC PERFORMANCE EVALUATION

Consider the communication system model illustrated in Fig. 2. We suppose there are N homogeneous and independent sources sharing the single-multiplexer and each source generates packets with average rate λ_i . The FEC coder for each source applies an interlaced Reed-Solomon code RS(n,k) to the packets from the source, which means for every block of K source information packets it creates an additional n-k parity packets to the network. The channel coding rate is given by $R_c = k/n$. Assume $P(j,n)$ denotes the block-error distribution, i.e., the probability that packets out of n are lost. Therefore, the expected number of lost packets within a block is

$$E[N_p] = \sum_{j=n-k+1}^n j * P(j, n)$$

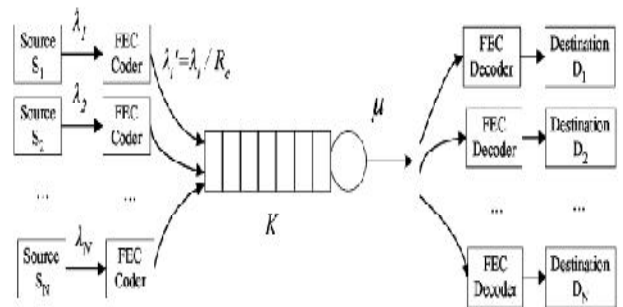


Fig.4 . Communication system model.

VI FEC PERFORMANCE WITH A SINGLE SOURCE

A. FEC Without Interleaving

We begin our analysis with the simplest case: there is only one user for the multiplexer ($N=1$). As illustrates, the key quantity in evaluating the residual packet-loss rate after FEC decoding is $P(j,n)$, the block-error distribution for an arbitrary number n of consecutive packets. In Cidon et al. propose a recursive algorithm to compute $P(j,n)$ for the finite buffer queue with Poisson arrivals and exponential service times, denoted as the $M/M/1/K$ queue. In order to analyze the packet losses for more general arrival patterns, in what follows we first describe the extension of the algorithm to the $G/M/1/K$ queue, i.e., the finite buffer queue with general i.i.d. interarrival times and exponential service times.

1) *Analysis of Block-Error Distribution:* Suppose there is only one source sharing the multiplexer ($N=1$) and the packet interarrival times are i.i.d. with arbitrary probability density Function $a(t)$.

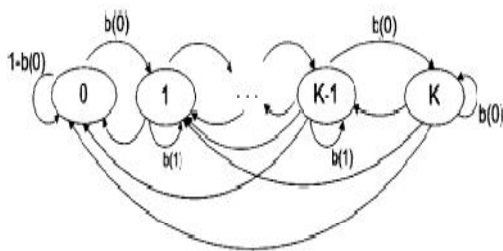


Fig. 5. State transitions of arrival-epoch system size.

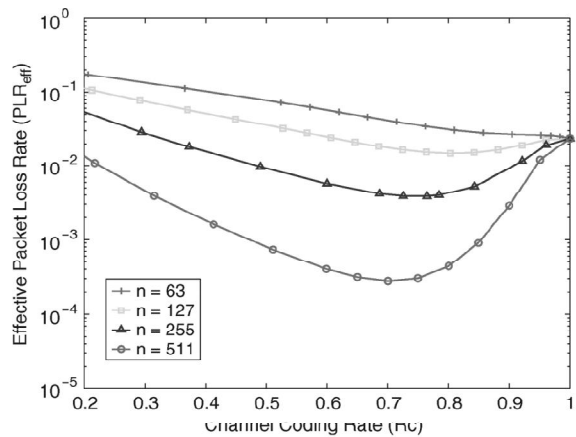


Fig. 6. Effective packet-loss rates; Poisson arrivals ($h = 1$), $\rho = 0.8$, $K = 10$, block size $n = 63, 127, 255, 511$.

2) *Numerical Examples:* Fig. 6 demonstrates the effective packet-loss rates PLR_{eff} according to computed with different coding block size $n=63,127,255$, and 511 as a function of coding rates $R_c=k/n$.

B. FEC With Block Interleaving

FEC performance is often limited by the bursty nature of typical packet-loss processes, and block interleaving techniques are frequently used to reduce the burstiness of the packet-loss processes in networks, thereby improving FEC performance. In this section, we analyze the efficacy of interleaving in reducing the burstiness of network packet-loss processes and in improving the FEC performance.

1) *Interleaving Operation:* The operation of block interleaving is illustrated in Fig:13. Before being transmitted into the network, packets are filled into an $M_1 * M_2$ Row wise.

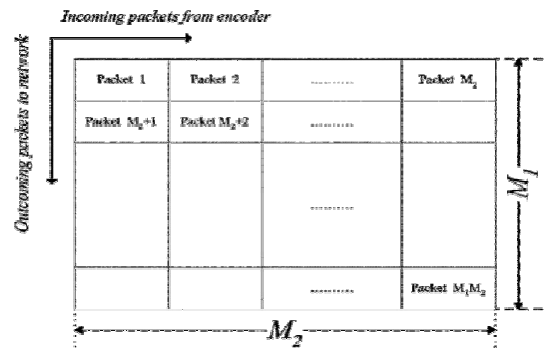


Fig. 7. Illustration of block interleaving operation (interleaving depth = M_1).

Fig. 7 shows the case of deterministic arrivals ($h=\infty$) with all other system parameters the same as in Fig. 6. Compared to Fig. 6, Fig. 5 shows that for more deterministic source arrivals an increased coding rate R_c is required to achieve the optimum performance. Both figures demonstrate that with interleaving the performance of FEC coding can be greatly improved, and interleaving with even larger depth can achieve increasingly improved performance.

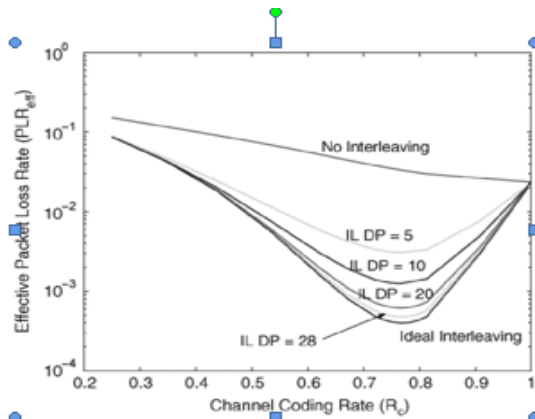


Fig: 8. Evaluate interleaving FEC performance poisson arrival

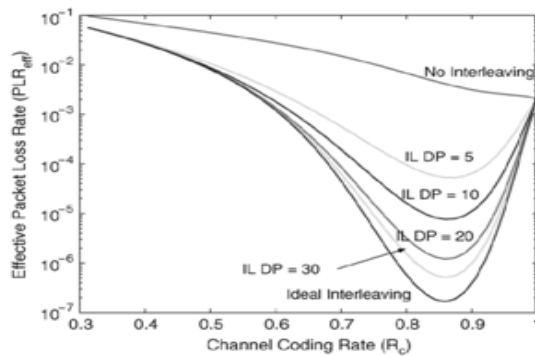


Fig: 9. Evaluate interleaving FEC performance deterministic arrival

IV. FEC PERFORMANCE WITH MULTIPLE SOURCES (N>1)

In Section III-A, we studied the FEC performance when there is only a single source using the multiplexer. In this section, we proceed to investigate FEC performance in case of multiple sources sharing the multiplexer. In order to facilitate the analysis, in this section we assume the packet arrival process seen by the multiplexer from each source is Poisson.

A. FEC Performance Without Interleaving

In order to evaluate the FEC performance for one of the N sources, the block-error distribution $P(j,n)$, for a single isolated source is required. Cidon et al. In this work, however, we describe a different method to compute $P(j,n)$ for the $N \cdot M/M/1/K$ queue, which can be extended easily to incorporate the analysis for interleaving in Section IV-B.

1) Analysis of Block-Error Distribution for a Single Source:

Assume the packets arriving at the single-multiplexer come from N independent sources: S_1, S_2, \dots, S_n , indicates that, for N homogeneous sources with a fixed overall load ρ , the loss process of a single source becomes less and less correlated with increasing N.

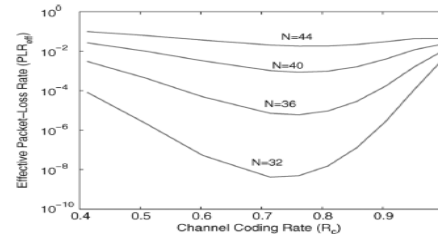


Fig. 10. FEC performance with N homogeneous sources; Poisson arrivals, load from each source fixed at $\rho_i=0.02$, $K=10$ block size $n=63$

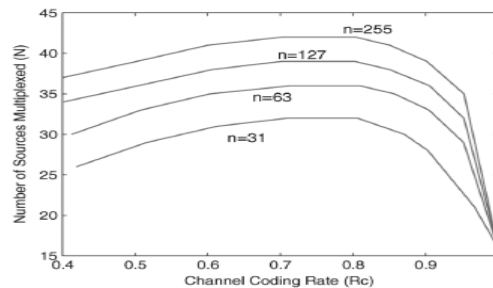


Fig. 11. Multiplexing gain achieved by FEC coding with different coding block sizes n; Poisson arrivals, the effective packet-loss rate fixed at $PLR_{eff}=10^{-6}$, load from each source fixed at $\rho_i=0.02$, $K=10$.

Now we study the FEC performance in improving the statistical multiplexing gain. As shown in Fig. 2, we suppose the FEC coder for each homogeneous source applies an $RS(n,k)$ code to the packets from the corresponding source coder. The channel coding rate remains $R_c = k/n$. As a result of the channel coding, the packet arrival rate into the single-multiplexer will increase to $\lambda^1 = \lambda_i/R_c$. We assume that the average load from each source is fixed while the total load $\rho = \lambda/\mu$ changes with varying N.

Fig. 6 demonstrates the FEC performance with different numbers of sources multiplexed, where the load from each source is fixed at $\lambda_i = 0.02$ with buffer size $K = 10$ and coding block size $n=63$. It shows that, with an increase in the number of sources N, the effective packet-loss rates increase due to the increased system load. Suppose now the load from each source is again $\lambda_i = 0.02$ and the required effective packet-loss rate is 10^{-6} . Fig. 20 demonstrates the maximum number of sources that can be multiplexed under these conditions. In

particular, it shows that, compared to the case where no coding is used ($R_c = 1$), with FEC coding the maximum N that can be multiplexed can be increased significantly provided that R_c is selected appropriately, and coding with larger block sizes can achieve even larger multiplexing gain.

B. FEC Performance With Interleaving

Now we suppose the packets from each homogeneous source are interleaved with the same interleaving depth before being transmitted into the network. The algorithm for computing the block-error distribution $P(j,n)$ for a single source can be extended to include the interleaving procedure, as provided in Section III-B. It can be expected that, compared to the case of a single source ($N=1$), the need for interleaving will be significantly reduced in a multiplexing environment ($N \geq 2$), due to the already reduced packet-loss correlation as a result of the natural interleaving effect of multiplexing. Fig. 11 shows the to the already reduced packet-loss correlation as a result of the natural interleaving effect of multiplexing. Fig. 11 shows the

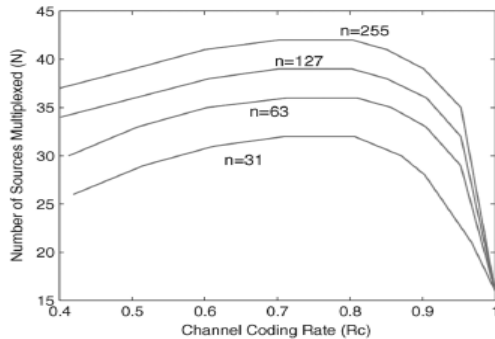


Fig. 12. Multiplexing gain achieved by FEC coding with different coding block sizes n ; Poisson arrivals, the effective packet-loss rate fixed at $PLR_{eff} = 10^{-6}$, load from each source fixed at $\rho_i = 0.02, K = 10$.

FEC performance with different interleaving depths, where the number of sources is $N = 3$ and the total system load is fixed at $\rho = 0.8$

(Scenario 1) with buffer size $K = 10$. As expected, when , in order to optimize the FEC performance, an interleaving depth $M \geq 10$ is required, while in Fig. 15 where $N=1$ an interleaving depth $M \geq 28$ is required. This point is further illustrated in Fig. 22, which demonstrates the interleaving depth required to approach the optimum FEC performance with different numbers of sources N

for a given total load $\rho=0.8$. It shows that when the number of sources N increases, the need for interleaving depth decreases, which means reduced latency associated with the interleaving/deinterleaving operation. The figure shows that, when $N \geq 14$, interleaving makes an insignificant difference in FEC performance, because in this case the packet-loss process of each source is nearly independent.

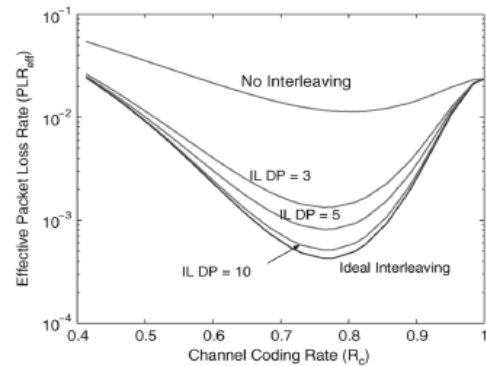


Fig. 13. Effect of interleaving on FEC performance with $N=3$ sources; Poisson arrivals, total load fixed at $\rho = 0.8, K = 10, n=63$.

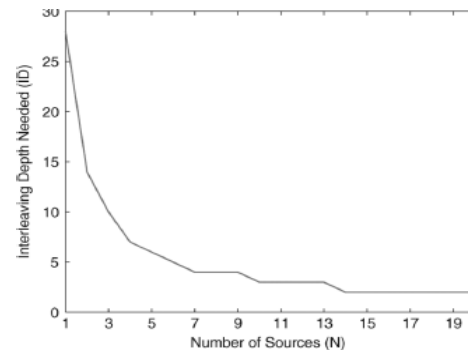


Fig. 14. Interleaving depth needed to approach maximum FEC performance versus the number of sources multiplexed n ; Poisson arrivals, total load fixed at $\rho = 0.8, K = 10, n=63$

V. POTENTIAL OF FEC AND AN INFORMATION-THEORETIC BOUND

In Fig. 17, we demonstrated that, with the same packet-loss rate requirement, FEC coding with a larger block size can support increased source traffic. However, the source traffic that can be supported is not unlimited because of the channel capacity limitation imposed by the single-multiplexer transport channel model. In what follows we develop an information-theoretic upper bound on the FEC performance based on the single-multiplexer network model. In this section, we only

consider the case of a single source ($N = 1$), although the approach can be extended to arbitrary N .

A. Channel Model for Packet Transmission Over Networks

Consider a channel model for packet transmission over a general packet-switched network. Assume a packet has m bits. It is either transmitted and received by the receiver, or is lost due to network congestion or buffer overflow. For a received packet, bit errors may be introduced. Then packet transmission over networks can be modeled for coding purpose in terms of serial bit-by-bit transmission of m -bit symbols either over a binary symmetrical channel (BSC) with crossover probability p

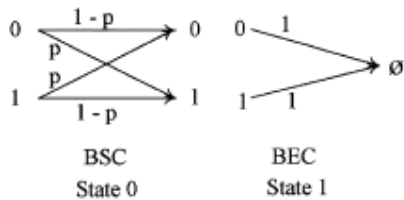


Fig. 15. Component channels of BIC corresponding to packet delivery and loss.

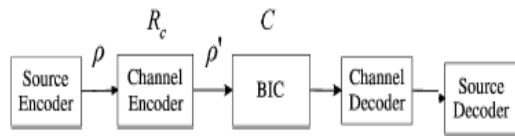


Fig. 16. Simplified communication system model.

(state 0) or over a binary erasure channel (BEC) (state 1), both of which are illustrated in Fig. 16 where ϕ is used to indicate the erasure symbol. A lost packet corresponds to the entire codeword symbol of m bits being erased, while a received packet means each of the m bits is sequentially transmitted over the BSC. This channel model belongs to the class of block interference channels (BIC), introduced by McEliece and Stark. Let $S \in \{0,1\}$ represent the state space of the BIC. If the state transitions are independent, then the Shannon capacity of the BIC is given as ,

$$C = E\{C_s\}; \text{ bits/transmission} \quad (1)$$

Where C_s is the capacity of the component channel $s \in S$, and the expectation is over the state space S . It follows that

$$C = (1-p) * (1-H(p)); \text{ bits/transmission,} \quad (2)$$

where p is the probability of being in the loss state and $H(p)$ is the binary entropy function,

$$H(p) = -p \log p - (1-p) \log (1-p); 0 \leq p \leq 1. \quad (3)$$

B. Information-Theoretic Bound on FEC Performance

Referring to Fig. 12, suppose the interleaving is ideal, and consequently the packet-loss process seen by the channel decoder is independent. If we consider the interleaver and the deinterleaver as components of the coding channel, then the channel, consisting of the interleaver, the single-multiplexer and the deinterleaver, can be modeled as a BIC with independent state transitions, as illustrated in Fig. 14. Here we consider only the packet losses caused by the buffer overflows, and assume no bit errors, i.e., the BSC crossover probability $p=0$. Let P_L be the packet loss rate of the single-multiplexer, so $p=P_L$. Then, from (4),

the capacity of the BIC is given by

$$C=(1-p)*(1-H(p))=1- P_L \quad (5)$$

Assume the source creates packets at rate λ and the packet service rate is μ . Then the normalized system load before coding is $\rho = \lambda/\mu$. The channel encoder applies channel coding (not necessarily RS codes) with coding rate R_c to the source traffic.

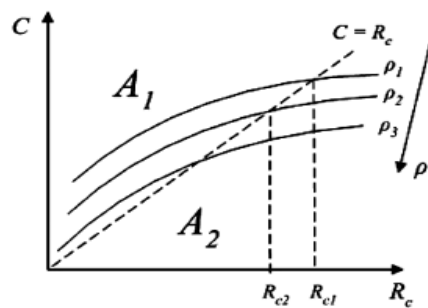


Fig.17. Schematic illustration of the functional relationship $C = 1 - f(\rho / R_c)$, for different values of ρ with $\rho_1 \leq \rho_2 \leq \rho_3$

Then the normalized system load after coding will increase to $\rho^1 = \rho/R_c$

Given the buffer size K , the average raw packet loss rate P_L depends only on the load ρ^1 as expressed by

$$P_L = f(\rho^1) = f(\rho / R_c) \quad (6)$$

where the function f can be determined by queueing analysis of the single-multiplexer model, which has been described in Section III-A1. From (5) and (6) we have,

$$C = 1 - P_L = 1 - f(\rho / R_c) \quad (7)$$

From (7), for a given load ρ , we can plot the functional relationship of C with R_c , as illustrated in Fig. 25. The channel coding theorem establishes that any rate less than the channel capacity can be supported with arbitrary low error probability. In other words, with regards to our model discussed here, as long as the channel coding rate R_c is smaller than the BIC capacity C , the source rate can be supported with arbitrarily high reliability. Therefore, in Fig. 17 the area above the line $C = R_c$, denoted by A_1 , where $R_c < C$, represents the source rates that can be supported with arbitrarily high reliability. The area below the line $C = R_c$, denoted by A_2 , represents the source rates that will inevitably incur some loss regardless of the channel coding scheme employed. For example, from Fig. 17 for the coding rate R_{c1} , the maximum source rate that can be supported with arbitrarily high reliability is ρ_1 . More generally, for the coding rate R_c , the maximum source rate that can be supported with arbitrarily high reliability is

$$\rho_{max} = \max\{\rho : R_c \leq 1 - f(\rho / R_c)\} \quad (8)$$

For example, for the M/M/1/K queue model, we have a closed-form expression for $f(\rho)$:

$$P_L = f(\rho^1) = (1 - \rho^1) (\rho 1)^k / 1 - (\rho^1)^{K+1} \quad (9)$$

From (31), (32) and (33), after some simplification, we have

$$\rho_{max} = \max\{\rho : R_c \leq [(R_c - \rho) * \rho^K] / [R^{k+1} - \rho^{k+1}]\} \quad (10)$$

Then, for a given buffer size K and choice of R_c , the maximum source load that can be supported with arbitrarily high reliability can be obtained from (10) by simple numerical search.

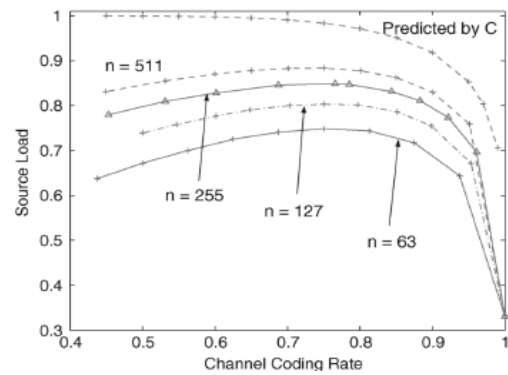


Fig. 18. The upper bound on the source loads ρ_{max} predicted by the channel capacity considerations, compared to the maximum source loads ρ_{max} that can be supported at a fixed effective packet-loss rate $PLR_{eff} = 10^{-5}$ using FEC coding; Poisson arrivals, ideal interleaving,

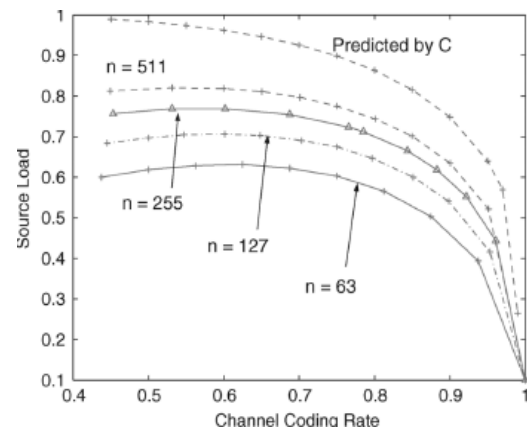


Fig. 19. The upper bound on the source loads ρ_{max} predicted by the channel capacity considerations, compared to the maximum source loads ρ_{max} that can be supported at a fixed effective packet-loss rate $PLR_{eff} = 10^{-5}$ using FEC coding; Poisson arrivals, ideal interleaving, $K = 5$. For more general cases, the maximum source load ρ_{max} can be obtained from (8).

Corresponding to Fig. 17 for the M/M/1/K model, Fig. 18 shows the upper bound on the source loads that can be supported as predicted by the preceding channel capacity considerations. It shows that with increasing coding block size the end-to-end performance achieved by FEC coding approaches that predicted by channel capacity. Fig. 19 shows the case of a smaller buffer ($K = 5$). The two figures indicate that, generally, the system with a larger buffer has a larger capacity. Note that in these two figures the capacity C is the capacity of the single-multiplexer combined with an ideal interleaver/deinterleaver, and not the capacity of the single-multiplexer itself. Actually, the capacity of the single-multiplexer channel can be greater than C

described here since the capacity of the memoryless interleaved channel is generally lower than the capacity of the original channel .

VI. CONCLUSIONS AND FUTURE

We have analyzed the efficacy of FEC in combating network packet losses based on a single-multiplexer network model and demonstrated that FEC has great potential in recovering the packet losses caused by congestion at a bottleneck node of a packet-switched network, provided that the coding rate

and other coding parameters are appropriately chosen. We developed a discrete-time Markov chain model to analyze the efficacy of interleaving in improving the FEC performance and determined how much interleaving depth is required for FEC to approach the optimum performance. We derived an upper bound on the end-to-end performance using FEC based on an information-theoretic methodology, which is useful in predicting source rates that can be supported with arbitrarily high reliability.

Despite the great potential of FEC coding in recovering network packet losses, the implementation complexity of FEC coding and the corresponding coding/decoding delay also need to be considered, which is an issue particularly important for real-time applications. One objective for future work is the analysis of the additional delay caused by the FEC coding, perhaps combined with interleaving/deinterleaving. Likewise, the application of FEC for network transport is limited by the time-varying and often uncertain error characteristics of the channel, which makes the appropriate choice of FEC coding rate difficult to determine. In real-world applications, FEC coders are required which can adapt the channel code rate to the time-varying channel conditions. This issue is also a topic for future work.

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