Computation of the largest Eigenvalues using Power method and Gerschgorin circles method

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***Abstract*— In this paper computation of largest eigenvalues has been presented using the Gerschgorin circles method. This is a graphical approach which takes less computation compared with the existing method.**

Keywords*— *Largest eigenvalues, Gerschgorin circles, Gerschgorin bound, system matrix

Introduction

The concept of computation of largest eigenvalue plays very important role. There exist several methods in literature to compute largest eigenvalues In [1] power method is used to compute the largest eigenvalues which takes many iterations.

In this paper , an attempt has been made to compute the largest eignvalue using the Gerschgorin circles. This method is applicable only when the bound on the right or left half of the s-plane are larger. Examples have been illustrated where the method cannot be applicable. This graphical technique takes fewer computation compared with the existing method.

I. EXISTING METHOD [1]

Consider the system matrix of order $(3x3)$

$$A = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \rightarrow (1)$$

Choose the initial vector $X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Then X1 $= A X_0 = \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ & & \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ \end{pmatrix}$

2 2 0 1 4

$$X_2 = A X_1 = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$X_3 = A X_2 = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$$

Continuing these iterations

$$X_4 = A X_3 = \begin{pmatrix} -4 \\ 0 \\ -4 \end{pmatrix} \quad X_5 = A X_3 = \begin{pmatrix} 12 \\ -14 \\ 4 \end{pmatrix}$$

$$X_6 = A X_5 = \begin{pmatrix} -28 \\ 30 \\ -4 \end{pmatrix} \quad X_7 = A X_6 = \begin{pmatrix} 60 \\ -62 \\ 4 \end{pmatrix}$$

$$X_8 = A X_7 = \begin{pmatrix} -124 \\ 126 \\ -4 \end{pmatrix} \quad X_9 = A X_8 = \begin{pmatrix} 252 \\ -254 \\ 4 \end{pmatrix}$$

Put $X=X_8$ $Y = X_0$ then

$$m_0 = X^T X = [-124\ \ 126\ \ -4]\begin{bmatrix}-124\\126\\4\end{bmatrix} = 31268$$

$$m_1 = X^T Y = [-124\ \ 126\ \ -4]\begin{bmatrix}252\\-254\\4\end{bmatrix} = -63268$$

$$m_2 = Y^T Y = [-252\ \ -254\ \ 4]\begin{bmatrix}252\\-254\\4\end{bmatrix} = 128036$$

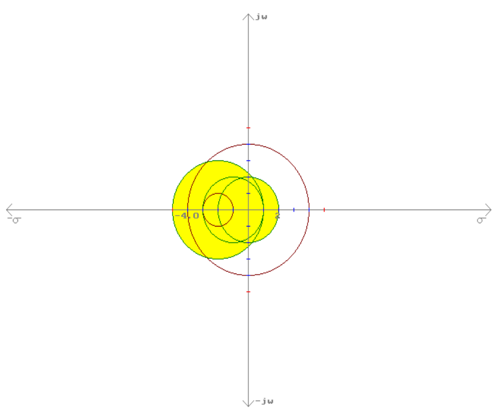
The largest eigenvalue is

$\lambda_{max} = m_{1/} m_2 = -63268/ 128036 = -2.0234105$ (9th iteration)

Total computation taken by the above method: 82

Gerschgorin circle method:

Gerschgorin circle of the matrix (1)



Gerschgorin bound [-4, 2]

Procedure: From the above figure we observe that Gerschgorin bound [-4 , 2] so to test the maximum eigenvalue we start with -4 .First we check whether or not the eigenvalue lie in [-4 , -3] by computing the determinant $(\lambda_i I - A)_{\lambda=-4}$ and determinant $(\lambda_i I - A)_{\lambda=-3}$ which implies that there is no eigenvalue in the interval [-4 , -3] .Then we check in the interval [-3 ,-2] and we found the determinant $(\lambda_i I - A)_{\lambda=0} = 0$ at $\lambda = -2$ .Hence the maximum eigenvalue is -2 .

Total computation: 16

From the above we observe that the Gerschgorin circle method takes less computation compared to the power method.

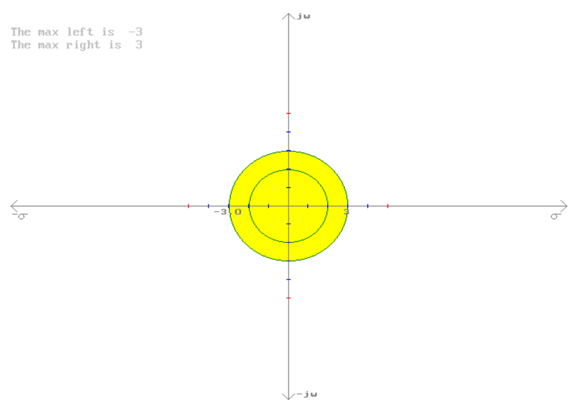
Conclusion: our method of computing eigenvalues via Gerschgorin circles works for all the examples when the Gerschgorin bound are unequal. We have presented the examples were the method does not work.

Counter example to the above method:

Consider the system matrix of order $(4x4)$

$$\begin{pmatrix}0 & -1 & 1 & 0\\-1 & 0 & 1 & 1\\1 & -1 & 0 & -1\\1 & 1 & -1 & 0\end{pmatrix} (4x4)$$

Gerschgorin circles of the above matrix are



Gerschgorin bound [-3 , 3]

The eigenvalues are

$\lambda_1 = -1.8136$

$\lambda_2 = 2.3429$

λ_3 = 0.4707

λ_4 = -1.0000

Incidentally we got the following result

(i) The $trace = 0$ and the bounds are equal.

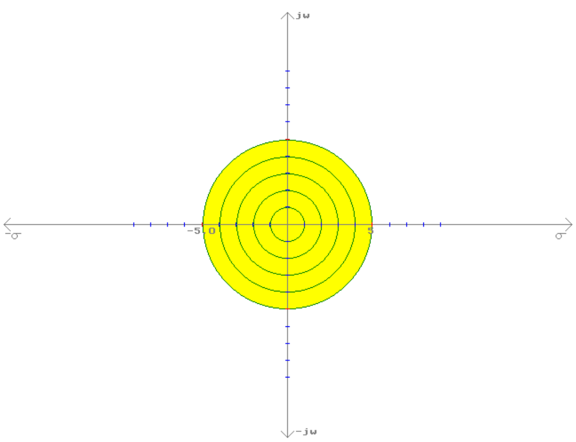
(ii) When $trace \neq 0$ and the bounds are unequal.

Case (i); When the $trace = 0$ and bounds are equal .Given all the centres of the Gerschgorin circles are at origin, this implies that there exists at least one eigenvalue on the positive real axis of s-plane. The eigenvalues may be complex conjugate eigenvalues with positive real part.

Consider the system matrix of order $(6x6)$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 \end{pmatrix}$$

Gerschgorin circles of the above matrix are



Gerschgorin bound [-5, 5]

The eigenvalues are

λ_1 = -2.4836

λ_2 = 1.5488

λ_3 = 0.5184

λ_4 = 0.0000

$\lambda_5 = -0.2077 + 0.6772\, i$

$\lambda_5 = -0.2077 - 0.6772\, i$

Conclusion: From the above example we observe that

(a) Centers of the Gerschgorin circles are at origin and hence $trace = 0$

(b) Gerschgorin bounds are equal on either side of the imaginary axis.

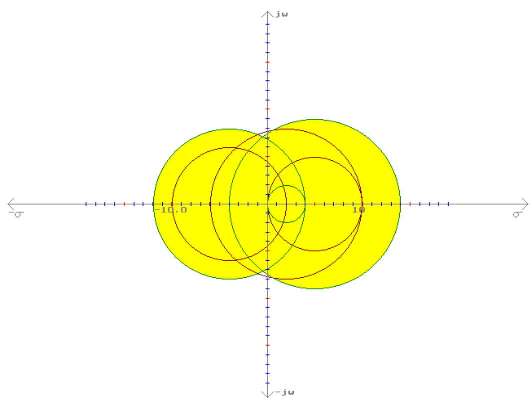
(c) There exist a positive real eigenvalue and complex conjugate pairs with negative real part.

Case (ii) : when $trace \neq 0$ and bounds are equal

Consider the system matrix of order $(3x3)$

$$\begin{pmatrix} 2 & -4 & -4 \\ 1 & -4 & -5 \\ -1 & 4 & 5 \end{pmatrix}$$

Gerschgorin circles of the above matrix are



Gerschgoin bound [-10, 10]

Eigenvalues of the above matrix are

λ_1 = 2.0000

λ_2 = 1.0000

λ_3 = 0.0000

All the eigenvalues are positive.

Hence finally we conclude for the above two cases there exist at least one eigenvalue (real or complex) on the right hand side of s-plane. Hence the systems are unstable .But we cannot decide about largest eigenvalue graphically in above cases.

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