Seasonal Time Series and Transfer Function Modelling for Natural Rubber Forecasting in India

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Abstract **— Time series analysis is a powerful tool to determine dynamic models aiming at defining and controlling most appropriate variables of a system. Transfer function model is one of the popular techniques in the time series modelling for forecasting. When there is an output series which is inclined by an input series, the objective of the transfer function modelling is to identify the role of input series in determining the variable of interest. In this paper, the Transfer Function model is fitted to the Natural Rubber production in India. The Transfer function Model has been used to identify a model and estimate parameters for forecasting of rubber production.**

Keywords **— Time series, Transfer function, Autoregressive Integrated Moving Average Model, Cross correlation Function.**

I. INTRODUCTION

In forecasting and analysis of time series data, it is well demonstrated that autoregressive integrated moving average (ARIMA), intervention and transfer function model are very effective in handling practical application. Modelling and forecasting of multi variable time series is to employ transfer function models. Transfer function models [3] can be regarded as extensions of classical regression models, and are useful in many applications.

Univeriate model uses a single dependent or output variable as a function of its own history and previous errors. Transfer function model is single or Multiple inputs that may possibly affect the system. The dynamic characteristics of a system are fully understood explicitly only though a transfer function model. The dynamic nature of the transfer function relationship lies in its ability to account for the instantaneous and lagged effects of an input variable on the output variable.

Bambang Widjanarkon Otok et al.,[5] used the Box Jenkins methodology to build Transfer function model for rainfall index data in Indonesia by comparing the forecast accuracy among ARIMA, ASTAR, Single – input Transfer function, and multiple input Transfer function models. Three locations of rain fall data at et Java are used as case study, i.e. Ngale, Karangjati, and Mantingan. In this research, Seasonal ARIMA as the appropriate type of rainfall index data is used. Khim et al.,[7] have proposed transfer function model to predict electricity prices based on both past electricity prices and demand and discussed the rational to build it. In this paper, the Transfer function model is fitted to the natural rubber production which is mostly influenced by sales. The dynamic relationship between the sales is studied through the Transfer function model. Finally the fitted Transfer Function model can also be used for forecasting. Transfer function [6] used the two techniques to accurately predict time series data of natural field latex prices. The time series forecast based on transfer function method was compared to neural networks model across the 2 periods ahead in the forecast horizon. The result of the study imply that neural networks forecasting method is a better alternative approach for predicting natural field latex prices.

II. DATA

Rubber is an amorphous, elastic material obtained from the latex or sap or various tropical plants. There are two types of rubber natural and synthetic. India is the fourth largest producer of natural rubber in the world and it is the fifth largest consumer. Though India is one of the leading producers of rubber it still imports rubber from other countries. Rubber-producing areas in India are divided into two zones traditional and non-traditional. Under traditional zone we have Kanyakumari in Tamil Nadu and some districts of Kerala whereas under non-traditional zone, rubber is produced in coastal regions of Karnataka, Goa, Andhra Pradesh, Orissa, some areas of Konkan regions of Maharashtra, Tripura, and Andaman and Nicobar Islands. Rubber production values (Tonnes) are recorded by the rubber industry using Monthly data for the period from January 1991 to December 2012 (264 months). The Natural Rubber production are collected from (www.induisttrialrubbergoods.com) for 264 months (January, 1991 to December, 2012).

III. METHODOLOGIES

A. ARIMA Model

One of time series models which is popular and mostly used is ARIMA model. Based on Autoregressive (AR) model shows that there is a relation between value in the present (Z_t) and values in the past (Z_{t-k}), added by random value. Moving Average (MA) model shows that there is a relation between a value in the present (Z_t) and residuals in the past (a_{t-k} with k=1, 2,…). ARIMA (p, d, q) model is a mixture of AR (p) and MA (q), with a non stationary data pattern and d differencing order. The form of ARIMA (p, d, q) is

$$\phi_p(B)(1-B)^d Z_t = \theta_q(B)a_t$$

Where p is AR model order, q is MA model order, d is Differencing order, and

$$\phi_p(B) = \left(1 - \phi_1 B - \phi_2 B^2 - . . . - \phi_p B^p\right)$$
$$\theta_q(B) = \left(1 - \theta_1 B - \theta_2 B^2 - . . . - \theta_q B^q\right)$$

Generalization of ARIMA model for a seasonal pattern data Which is written as ARIMA (p, d, q)(P,D,Q)^s ,is

$$\phi_p(B)\phi_p\left(B^s\right)(1-B)^d\left(1-B^s\right)^D Z_t = \theta_q(B)\phi_Q\left(B^s\right)a_t$$

Where s is seasonal period

$$\phi_p\left(B^s\right) = \left(1-\phi_1 B^s - \phi_2 B^{2s} - . . . - \phi_p B^{ps}\right) \quad and$$
$$\phi_Q\left(B^s\right) = \left(1-\phi_1 B^s - \phi_2 B^{2s} - . . . - \phi_Q B^{Qs}\right)$$

B. Transfer Function

Transfer function model is different from ARIMA model. ARIMA model is univerate time series model but transfer functions multivariate time series model. This means that ARIMA model relates the series only to past. Besides the past series, transfer function model also relates the series to other time series. Transfer function model can be used to model single output and multiple output system, In the case of single output model, only one equation is required to describe the system, It is referred to as a single equation transfer model. A multiple output transfer function model is referred to as a multi equation transfer function model or a simultaneous transfer function model function (STF) model. A more complete description of modelling and forecasting using multi equation model can be found. A single equation transfer function model may contain more than one input variable as in multiple regression models.

The single input transfer function model it,

$$Z_t = c + \frac{\omega_s(B)}{\delta_r(B)} B^b X_t + N_t$$
$$N_t = \frac{\theta(B)}{\phi(B)} a_t$$

Where,

$$\omega_{s(B)} = \omega_0 + \omega_1 B + \omega_2 B^2 + . . . + \omega_s B^s$$
$$\delta_r(B) = 1 + \delta_1 B + \delta_2 B^2 + . . . + \delta_r B^r$$
$$\theta(B) = 1 - \theta_1 B - \theta_1 B^2 - . . . - \phi_1 B^q$$
$$\phi(B) = 1 - \phi_1 B - \phi_1 B^2 - . . . - \phi_1 B^p$$

Multi Input Transfer Function Model

$$Z_t = C + \frac{\omega_{s1}(B)}{\delta_{r1}(B)} B^b X_{1t} + \frac{\omega_{s2}(B)}{\delta_{r2}(B)} B^b X_{2t} + . . . + \frac{\omega_{sm}(B)}{\delta_{rm}(B)} B^b X_{mt} + N_t$$

Where the rational transfer function $\frac{\omega si(B)}{\delta ri(B)}$ B^{bi}, for each input variable has the form in single transfer function model.

1) Identification of a model describing η_t and of a final transfer function model: It is necessary to check whether the preliminary model is adequate by analysing the residuals (η_t) with the values of the input x_t. The model describes η_t is determined by the following equation:

$$\eta_t = \frac{\hat{\phi}_{pn}(B)}{\hat{\phi}_{qn}(B)} \varepsilon_t$$

Where η_t is a disturbance term that follows an ARIMA model, Hence, an appropriate final transfer function model is of the form:

$$Z_t = \mu + \frac{C\varpi(B)\eta\delta}{\delta(B)} B^b Z_t^{(x)} + \frac{\hat{\phi}_{pn}(B)}{\hat{\theta}_{qn}(B)} \varepsilon_t$$

1.1) Prewhitening of x_t and y_t : Once an appropriate model describing x_t can be identified the relationship between x_t and y_t will be estimated. The prewhitened x_t and y_t values can be calculated by equations respectively:

$$\alpha_t = \frac{\hat{\phi}_{p^{(x)}}(B)}{\hat{\theta}_{q^{(x)}}(B)} z_t^{(x)} \; for \quad x_t \; x$$

$$\beta_t = \frac{\hat{\phi}_{p^{(x)}}(B)}{\hat{\theta}_{q^{(x)}}(B)} z_t^{(x)} \; for \quad y_t$$

Where $Z_t^{(x)}$ and Z_t represent input and output data that are transformed to be stationary form.

1.2) Calculation of the sample cross correlation function (CCF) and identification of a preliminary transfer function model : In order to identify a preliminary transfer function model describing the relationship between y_t and x_t, the sample cross correlation function (CCF) between the α_t value and the β_t value must be computed from the following equation:

$$r_k\left(\beta_{t,\alpha_t}\right)=\frac{\sum_{t=1}^{n-k}(\alpha_t-\bar{\alpha})(\beta_{t-k}-\bar{\beta})}{\sqrt{\sum_{t-1}^{n}(\alpha_t-\bar{\alpha})\sum_{t-1}^{n}(\beta_{t-\hat{\beta}})^2}}$$

The general preliminary transfer function model is computed as:

$$Z_t=\mu+\frac{C\varpi(B)\eta\delta}{\delta(B)}B^b Z_t^{(x)}+\frac{\hat{\phi}_{pn}(B)}{\hat{\theta}_{qn}(B)}\varepsilon_t$$

In selecting the form of the model the value of b, r and s must be determined from the correlogram of $r_k(\beta_t,\alpha_t)$. The valued of b is the number of periods before the input data (x_t) begins to influence output data (Y_t). It is equal to the lag where the first spike in the SCC is encounted or the number of weights that are not significantly from zero. The value of r represent the number its own past value z_t. The value of s represent the number of past $z_t^{(x)}$ value influencing z_t .

2) Parameter Estimation: In step2 after all function of the model has been structured the parameter of these functions will be estimated. Good estimators of the parameters can be found using least squares by assuming that those data are stationary.

2.1) Transfer function Order: For numerator and denominator components the value represents the maximum order. Order 0 always included for numerator components. Order 2 numerator the model include order 2,1 and 0.Order 3 denominator , the model include order 3,2and 1.The numerator under of the transfer function specifies which previous values from the selected independent (predictor) series are used to predict current values of the dependent series. The denominator order of the transfer function specifies how deviations from the series mean for previous value of the selected independent (predictor) series are used to predict current value of the dependent series.

Two method have been suggested for removing autocorrelation before calculating the CCF

- Pre-whiten (filter) the series recommended by Box and Jenkins.
- Fit separate ARIMA models to x_t and y_t and then Calculate the cross correlation function based on the residuals.

The first method is more widely used although the issue is not completely resolved; pre whitening involves five conceptual steps.

- First difference each series until it is stationary about its mean.

$$\omega_t=(1-B)^d X_t$$
$$Z_t=(1-B)^d Y_t$$

The degree of differencing need not be the same for the X_t and Y_t series.

- Second ,fit an ARIMA model to the differenced predictor series ω_t

$$\omega_t=\frac{\theta(B)}{\phi(B)}a_t$$

- Third, use the inverse of the model to filter ω_t (leaving the residual a_t)

$$a_t=\frac{\phi(B)}{\theta(B)}\omega_t$$

- Fourth, use the same inverse to the filter ωt (or pre whiten) Z_t. Yielding the pre whitened b_t

$$b_t=\frac{\phi(B)}{\theta(B)}Z_t$$

- Fifth, cross correlation the residuals a_t and the pre whitened b_t . This CCF reflects the impulse weight and therefore may be used to identify transfer function. The second method of removing autocorrelation is to fit separate ARIMA model to the predictor and output series and then filter each series alone to identify the transfer function.

3) Diagnostic checking: A diagnostic checking is employed to validate the model assumption and to check whether the model is adequate. It is necessary to do diagnostic checking even if the selected model may perform to be the best among others. It checks whether the hypothesis made on the residuals are true or not. These residuals must be a white noise series. Zero mean and constant variance, uncorrelated process and normal distribution. These requirements can be investigated by inspecting the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the residuals and taking tests for randomness such as Ljung – Box statistics.

4) Forecasting with transfer function: If the hypotheses on the residuals from step 3 are satisfied, the forecast prices of the finals model are then computed and compared the result with the test data.

- Transfer function model which are extension of Familiar linear regression models have been widely used in various field of research.
- Transfer function model can be used to study the Dynamic interrelationship among the variable in an economic system.
- The function identification method can be used in the Same manner no matter if the transfer function model has single input or multiple input variables. This method is more practical and easier to use then the cross correlation function.
- Transfer function models can be used to model can be used to model only one equation is required to describe the system. It is referred is required to

describe the system. It is referred to as a single equation transfer function model.

- A multiple output transfer function model is referred to as a multi equation transfer function model or a simultaneous transfer function model.
- The transfer function only present if independent Variables are specified.

C) Seasonal Transfer function: Seasonal Transfer function notation for i^{th} predictor time series $X_{i,t}$ with seasonal factor is

$$\lfloor Dif(d_i)(D_i)_s \, lag(k_i) N(q_i)_s / D(p_i)(p_i)_s \rfloor$$

where,

D_i – is the seasonal order of the differencing for the i^{th} predictor time series.

Pi – is the seasonal order of the denominator for the i^{th} predictor time series.

Q_i – is the seasonal order of the numerator for the k^{th} predictor time series

S – is the length of the seasonal cycle.

The mathematical notation used to describe a seasonal transform function is.

$$\psi_i(B) = \frac{\omega i(B)\omega s,i(Bs)}{\delta i(B)\delta s,i(Bs)}(1-B)^{di}\left(1-B^s\right)^{Di} B^{ki}$$

where,

$\delta_{s,i}\left(B^S\right)$ is the denominator seasonal polynomial of the transfer function for the i^{th} predictor time series

$$\delta_{s,i}\left(B^S\right) = 1 - \delta_{s,i,1}B - \ldots - \delta_{s,i,pi}B^{spi}$$

$\omega_{s,i}\left(B^S\right)$ is the numerator seasonal polynomial of the transfer function for the i^{th} predictor time series

$$\omega_{s,i}\left(B^S\right) = 1 - \delta_{s,i,1}B - \ldots - \omega_{s,i,Qi}B^{sQi} \quad \omega_{s,i}(B^s)$$

D) Result and Discussion: Transfer function method is a dynamic regression model which allows the explanatory variable to be included. The main objective of the model is to predict what happen to the forecast variable or output time series, called y_t, if the explanatory variable or input time series ,called x_t changes. Let x_t and y_t represent input and output data of transfer function model, respectively. These time series data consist of n observation of basis of the transfer function approach to modelling time series consists of four steps. The appropriate transfer function model will be identified. It is assumed that input and output time series must be both stationary. If not it is necessary to transform those data into stationary form. The plot graph of both the time series of rubber production and sales were examined and these shows that the seasonal time series of rubber were not stationary.

$$\psi_i(B) = 14.226 + \frac{0.478 * 0.943}{0.860 * 0.985}(1-B)\left(1-B^{12}\right)B^{12}$$

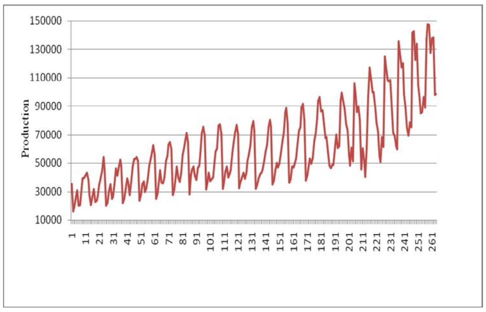

Fig. 1 Time series plot of actual rubber production in India

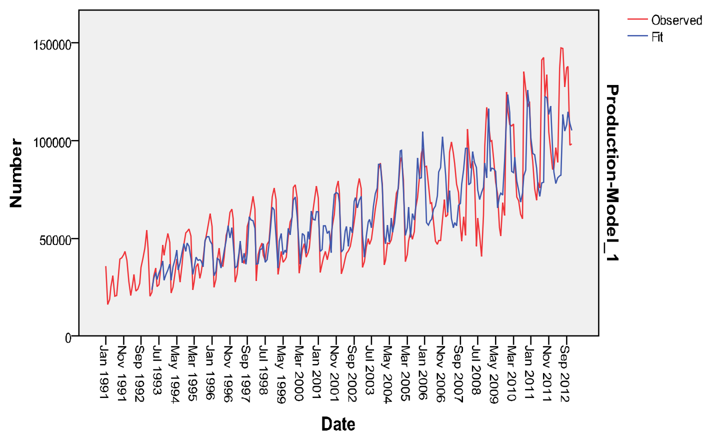

Fig. 2 The plot of Actual production against Forecast production by Seasonal Transfer function $(0,1,1)(1,1,0)_{12}$ Method

TABLE 1

TRANSFER FUNCTION MODEL AND MODEL STATISTIC

Fit Statistic	Mean
Stationary R- square	.720
R- Squared	.669
RMSE	16199.792
MAPE	18.235
MAE	11245.690
BIC	19.500

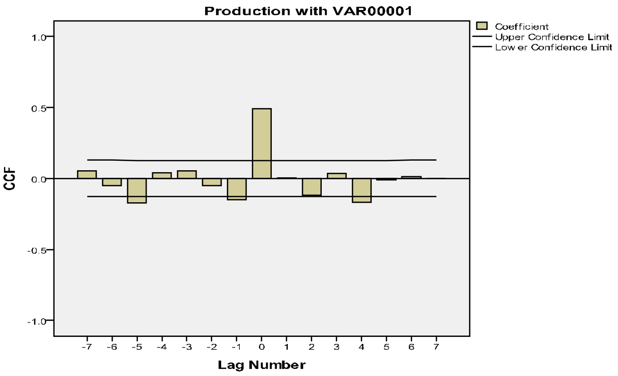

Fig. 3 The sample cross correlation function (SCC) and model statistic

TABLE 2
PARAMETER ESTIMATION OF TRANSFER FUNCTION MODEL

	TF	Estimate	SE	t	Sig
Production	Constant	14.22	4.149	3.428	.001
	Delay	12			
	Numerate Lag0	.478	.048	10.00	.000
Sales	Difference	1			
	Denominater lag 1	.860	.021	41.66	.000
	Numerater seasonal lag 1	.943	.031	30.64	.000
	Denominator seasonal lag 1	.985	.020	50.19	.000

IV CONCLUSION

In this paper, we have presented efficient techniques to accurately predict time series data of natural rubber production in India. The time series forecast based on transfer function method was compared to production and sales in the forecast horizon. The forecasted accuracy measure of the identified Transfer Function model is 19.500 which is small. From a detailed analysis of the numerical results, it can be concluded that the quality of predictions using the proposed technique is considerably good compared with other standard time series models when output series are influenced by input series.

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