Database Based Validation of Union of Two Multigranular Rough Sets

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Abstract**—** Most of the traditional tools for undertaking modeling, reasoning and other computing are found not only crisp but also highly deterministic and more precise in character which usually limits their applicability in real life situations led to the extension of the concept of crisp sets so as to model imprecise data and hence enhance their modeling capacity. One such method used to capture impreciseness was carried out by Pawlak who introduced the idea of rough sets, which is found to be an excellent tool to capture impreciseness in data. Several extensions have been made in different directions in order to improve the modeling capacity of the basic rough sets. One among such extension is rough set model based on multigranulations. Several fundamental properties of these types of rough sets have been studied . Pawlak introduced the types of rough sets in as an interesting characterization of rough sets by employing the ideas of lower and upper approximations of rough sets. There are two different ways of characterizing rough sets; the accuracy coefficient and the topological characterization introduced through the notion of types. As referred by Pawlak himself, in general rough sets by knowing the accuracy of a set, we were still unable to tell exactly its topological structure and also the present knowledge about the topological structure of the set gives no information about its accuracy. Therefore in practical applications of rough sets we combine both kinds of information about the borderline region, that is of the accuracy measure as well as the information about the topological classification of the set under consideration. Keeping this in mind, Tripathy and Mitra [9] have studied the types of rough sets by finding out the types of union and intersection of rough sets of different types. Later Raghavan et al have extended these results to the multigranular context in [12]. In this work I provided the database based validated results for the carried out results.

Index Terms**—** Rough sets, multigranular rough sets ,database , topological property.

I. Introduction

The basic assumption of rough set theory is that how the knowledge possessed by the human about a universe helps to classify the objects. Both classifications of a universe and equivalence relations defined on it are known to be interchangeable notions. For certain mathematical reasons equivalence relations were considered by Pawlak to define rough sets. A rough set is represented by a pair of crisp sets, called the lower approximation and upper approximation. The lower approximation consists of only certain elements where as upper approximations comprise of all possible elements with respect to the available information which was defined over the equivalence relations. Several extensions have been made in different directions in order to improve the modeling capacity of the basic rough sets. From the point of view of granular computing, the classical rough set theory was researched by a single granulation. The basic rough set model has been extended to rough set model based on multigranulations (MGRS) in [10], where the set approximations are defined by using multiple equivalence relations on the universe. Incomplete rough set model based on multigranulations was introduced in [11] by taking multiple tolerance relations instead of multiple equivalence relations. Several fundamental properties of these types of rough sets have been studied

[16, 17, 19]. Pawlak introduced the types of rough sets in [12] as an interesting characterization of rough sets by employing the ideas of lower and upper approximations of rough sets. There are two different ways of characterizing rough sets; the accuracy coefficient and the topological characterization introduced through the notion of types. As referred by Pawlak himself [8], in general rough sets by knowing the accuracy of a set, we were still unable to tell exactly its topological structure and also the present knowledge about the topological structure of the set gives no information about its accuracy. Therefore in practical applications of rough sets we combine both kinds of information about the borderline region, that is of the accuracy measure as well as the information about the topological classification of the set under consideration.

Definition 2.2.1: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, X $\subseteq$ U and $R, S \in \mathbf{R}$. We define the optimistic multi-granular lower approximation and optimistic multi-granular upper approximation of X with respect to R and S in U as

$$\underline{R+S}\,X = \{x \mid [x]_R \subseteq X \text{ or } [x]_S \subseteq X\}$$

and

$$\overline{R+S}\,X = \sim(\underline{R+S}(\sim X)).$$

Definition 2.2.2: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, X $\subseteq$ U and $R, S \in \mathbf{R}$. We define the pessimistic multi-granular lower approximation and pessimistic multi-granular upper approximation of X with respect to R and S in U as

$$\underline{R*S}\,X = \{x \mid [x]_R \subseteq X \text{ and } [x]_S \subseteq X\}$$

and $\overline{R*S}\,X = \sim(\underline{R*S}(\sim X))$.

II. TOPOLOGICAL PROPERTY OF BASIC ROUGH SETS

An interesting characterization of rough sets was introduced by Pawlak, namely the topological characterization or classification of rough sets [8]. This topological characterization is found to be an additional one to the characterization of rough sets by means of numerical values in the form of accuracy coefficients. While differentiating the topological characterization and accuracy coefficient Pawlak expressed that "The accuracy coefficient expresses how large the boundary region of the set is, but says nothing about the structure of the boundary, whereas the topological classification of rough sets gives no information about the size of the boundary region but provides us with some insight as to how the boundary region is structured" [11]. In general, topological properties of sets deal with the internal structures of sets. The following four types were defined by the Pawlak.

There are four different kinds of rough sets. These are defined as follows:

Type 1: If $\underline{R}X \neq \phi$ and $\overline{R}X \neq U$ then we say that X is roughly R-definable.

Type 2: If $\underline{R}X = \phi$ and $\overline{R}X \neq U$ then we say that X is internally R-undefinable.

Type 3: If $\underline{R}X \neq \phi$ and $\overline{R}X = U$ then we say that X is externally R-undefinable.

Type 4: If $\underline{R}X = \phi$ and $\overline{R}X = U$ then we say that X is totally R-undefinable.

In terms of their types a study was recently made by Tripathy and Mitra [9] and obtained some interesting properties on intersection and union of rough sets of different types for single granular rough sets (Basic rough sets).

III. ON SOME TOPOLOGICAL PROPERTIES OF MULTIGRANULAR ROUGH SETS

The topological properties of rough sets was introduced by Pawlak in terms of their types was recently studied by Tripathy et al to find the types of union and intersection of such sets and also complement of one such set for single granular rough sets. In this work we extended these results to the multigranulation context in [12] and as a result we have obtained the following types of multigranulation rough sets based on topological view.

Type-1: If $\underline{R+S}X \neq \phi$ *and* $\overline{R+S}X \neq U$ then we say that X is roughly R+S-definable.

Type-2: If $\underline{R+S}X = \phi$ *and* $\overline{R+S}X \neq U$ then we say that X is internally R+S- definable.

Type-3: If $\underline{R+S}X \neq \phi$ *and* $\overline{R+S}X = U$ then we say that X is externally R+S-definable.

Type-4: If $\underline{R+S}X = \phi$ *and* $\overline{R+S}X = U$ then we say that X is totally R+S-definable.

3.1 IMPORTANT OBTAINED RESULTS

3.1.1.UNION OF TWO MULTIGRANULAR ROUGH SETS

We shall provide an example to show that for two multigranular rough sets of type 1, the union can be of type 1 or type 3.

EXAMPLE

Let

$$U / P = \{\{e_1, e_2, e_7\}, \{e_4, e_5\}, \{e_3, e_6, e_8\}\}$$

$$U / Q = \{\{e_2, e_3, e_4, e_5\}, \{e_1, e_7, e_8\}, \{e_6\}\}$$

$$X = \{e_4, e_5\} \sim X = \{e_1, e_2, e_3, e_6, e_7, e_8\}$$

$$\underline{P+Q}(X) = \{e_4, e_5\} \neq \phi$$

$$\overline{P+Q}(X) =\sim (\underline{P+Q}(\sim X)) =\sim (\{e_1, e_2, e_3, e_6, e_7, e_8\}) = \{e_4, e_5\} \neq U$$

So X is of Type - 1.

$$Y = \{e_1, e_6\}$$

$$\sim Y = \{e_2, e_3, e_4, e_5, e_7, e_8\}$$

CASE 1

$$\underline{P+Q}(Y) = \{e_6\} \neq \phi$$

$$\overline{P+Q}(Y) =\sim (\underline{P+Q}(\sim Y)) =\sim (e_2, e_3, e_4, e_5) = \{e_1, e_6, e_7, e_8\} \neq U$$

So Y is of Type - 1.

$$X \cup Y = \{e_1, e_4, e_5, e_6\}$$

$$\sim (X \cup Y) = \{e_2, e_3, e_7, e_8\}$$

$$\overline{P+Q}(X \cup Y) =\sim (\underline{P+Q}(\sim (X \cup Y)) =\sim (\underline{P+Q}(\{e_2, e_3, e_7, e_8\}) =\sim (\phi) = U$$

$$\underline{P+Q}(X \cup Y) = \{e_4, e_5\} \neq \phi$$

X ∪ Y is of Type - 3

CASE 2

The following example is used to show the second case for multigranular rough set where both X and Y are of Type 1 and its union is also of type 1. The equivalence relation P and

Q remains same here.

$$X = \{e_3, e_6, e_8\}$$

$$\sim X = \{e_1, e_2, e_4, e_5, e_7\}$$

$$\underline{P+Q}(X) = \{e_3, e_6, e_8\} \neq \phi$$

$$\overline{P+Q}(X) =\sim (\underline{P+Q}(\sim X)) =\sim (e_4, e_5) = \{e_1, e_2, e_3, e_6, e_7, e_8\} \neq U$$

So X is of Type - 1.

$$Y = \{e_6\}$$

$$\sim Y = \{e_1, e_2, e_3, e_4, e_5, e_7, e_8\}$$

$$\underline{P+Q}(Y) = \{e_6\} \neq \phi$$

$$\overline{P+Q}(Y) =\sim (\underline{P+Q}(\sim Y)) =\sim (e_1, e_2, e_3, e_4, e_5, e_7, e_8) = e_6 \neq U$$

So Y is of Type - 1.

$X \cup Y = \{e_3, e_6, e_8\}$

$\sim (X \cup Y) = \{e_1, e_2, e_4, e_5, e_7\}$

$\underline{P+Q}(X \cup Y) = \{e_3, e_6, e_8\} \neq \phi$

$\overline{P+Q}(X \cup Y) = \sim (\underline{P+Q}(\sim (X \cup Y))$

$= \sim (\underline{P+Q}(\{e_1, e_2, e_4, e_5, e_7\})) = \sim (e_1, e_2, e_4, e_5, e_7) = \{e_3, e_6, e_8\} \neq U$

X ∪ Y is of Type - 1

The following example is used to show for the multigranular rough set where X is of type 1 and Y is of type 3 and its union is of type 3.

$U / P = \{\{e_1, e_7\}, \{e_2, e_3, e_4, e_5, e_6, e_8\}\}$

$U / Q = \{\{e_1, e_2\}, \{e_3, e_4, e_5\}, \{e_6, e_7, e_8\}\}$

$X = \{e_1, e_2, e_6, e_8\}$

$\sim X = \{e_3, e_4, e_5, e_7\}$

$\underline{P+Q}(X) = \{e_1, e_2\} \neq \phi$

$\overline{P+Q}(X) = \sim (\underline{P+Q}(\sim X)) = \sim (e_3, e_4, e_5) = \{e_1, e_2, e_6, e_7, e_8\} \neq U$

So X is of Type - 1.

$Y = \{e_1, e_2, e_4, e_8\}$

$\sim Y = \{e_3, e_5, e_6, e_7\}$

$\underline{P+Q}(Y) = \{e_1, e_2\} \neq \phi$

$\overline{P+Q}(Y) = \sim (\underline{P+Q}(\sim Y)) = \sim (\phi) = U$

So Y is of Type - 3

$X \cup Y = \{e_1, e_2, e_4, e_6, e_8\}$

$\sim X \cup Y = \{e_3, e_5, e_7\}$

$\overline{P+Q}(X \cup Y) = \sim (\underline{P+Q}(\sim (X \cup Y))$

$= \sim (\underline{P+Q}(e_3, e_5, e_7)) = \sim (\phi) = U$

$\underline{P+Q}(X \cup Y) = \{e_1, e_2, e_4, e_6, e_8\} \neq \phi$

X ∪ Y is of Type - 3

The following example is used to show for the multigranular rough set where X is of type 1 and Y is of type 4 and its union is of type 3.

$U / P = \{\{e_1, e_7\}, \{e_2, e_3, e_4, e_5, e_6, e_8\}\}$

$U / Q = \{\{e_1, e_2\}, \{e_3, e_4, e_5\}, \{e_6, e_7, e_8\}\}$

$X = \{e_1, e_2, e_6, e_8\}$

$\sim X = \{e_3, e_4, e_5, e_7\}$

$\underline{P+Q}(X) = \{e_1, e_2\} \neq \phi$

$\overline{P+Q}(X) = \sim (\underline{P+Q}(\sim X)) = \sim (e_3, e_4, e_5) = \{e_1, e_2, e_6, e_7, e_8\} \neq U$

So X is of Type - 1

$Y = \{e_1, e_3, e_8\}$

$\sim Y = \{e_2, e_4, e_5, e_6, e_7\}$

$\underline{P+Q}(Y) = \phi$

$\overline{P+Q}(Y) = \sim (\underline{P+Q}(\sim Y)) = \sim (\phi) = U$ So Y is of Type - 4

$X \cup Y = \{e_1, e_2, e_3, e_6, e_8\}$

$\sim X \cup Y = \{e_4, e_5, e_7\}$

$\underline{P+Q}(X \cup Y) = \{e_1, e_2, e_3, e_6, e_8\} \neq \phi$

$\overline{P+Q}(X \cup Y) = \sim (\underline{P+Q}(\sim (X \cup Y))) = \sim (\phi) = U$

X ∪ Y is of Type - 3

The following table summarizes the above discussed results in the tabular form.

		Type of Y with respect to P+Q			
Type of X with respect to P+Q		T-1	T-2	T-3	T-4
	T-1	T-1 / T-3	T-1 / T-3	T-3	T-3
	T-2	T-1 / T-3	T-1/T-2/T-3/T-4	T-3	T-3 / T-4
	T-3	T-3	T-3	T-3	T-3
	T-4	T-3	T-3 / T-4	T-3	T-3 / T-4

Table 2.5.1 Type of X $\bigcup$ Y with respect to P+Q

3.1.2 VALIDATION

Using the table given in the first chapter we computed four equivalence relations. The first equivalence relation is grouped division wise, the second equivalence relation is based on the present position or grade of the faculty members. The third and fourth equivalence relations are based on highest degree and native state of faculty respectively.

Let

$$\bigcup = \{sam, smith, jacob, shyam, john, keny, lakman, pretha, peter, albert, linz, sita, roger, mishra, williams, fatima, ram, hari, martin, mukherjee\}$$

$$\bigcup / HighestDegree = \{\{Shyam, albert, mishra, martin, jacob\}, \{sam, john\}, \{sita, fatima\}, \{ram, peter, roger, hari, smith, keny, linz, williams, lakman, mukherjee, preetha\}$$

$$\bigcup / Nativestate = \{\{sam, shyam, roger, mishra, williams\}, \{ram, peter\}, \{hari, smith, linz\}, \{peter, john, lakman, fatima, mukherjee\}, \{keny, martin, jacob, sita, pretha\}\}$$

We shall provide an example to show that for two multigranular rough sets of type 1, the union can be of type 1 or type 3

Let

U / P be the U / Highest Degree
U / Q be the U / Native State

Faculty Name	**Division**	**Grade**	**Highest Degree**	**Native State**
Sam	NW	AP	M.C.A	Tamil Nadu
Ram	IS	Pr	Ph.D	Andhra Pradesh
Shyam	SE	APJ	M.sc.,	Tamil Nadu
Peter	AI	ASP	Ph.D	Tamil Nadu
Roger	ES	P	Ph.D	Tamil Nadu
Albert	AI	APJ	M.Sc.,	Andhra Pradesh
Mishra	ES	APJ	M.Sc.,	Tamil Nadu
Hari	IS	SP	Ph.D	Orissa
John	SE	AP	M.C.A	West Bengal
Smith	NW	ASP	Ph.D	Orissa
Linz	AI	SP	Ph.D	Orissa
Keny	SE	P	Ph.D	Karnataka
Williams	ES	ASP	Ph.D	Tamil Nadu
Martin	IS	APJ	M.Sc.,	Karnataka
Jacob	NW	APJ	M.Sc.,	Karnataka
Lakman	SE	ASP	Ph.D	West Bengal
Sita	AI	AP	M.Tech	Karnataka
Fatima	ES	AP	M.Tech	West Bengal
Mukerjee	IS	SP	Ph.D	West Bengal
Preetha	SE	SP	Ph.D	Karnataka

The database which is stated above is used for the validation. Here NW indicates networks, SE indicates Software engineering, AI indicates Artificial Intelligence, ES indicated Embedded Sysems, IS indicated Information Systems. Similarly AP indicates Assistant Professor, APJ indicates Assistant Professor (Junior), ASP indicates Associate Professor, SP indicates Senior Professor, Pr indicates Professor.

In the following example

Example

X = {sam, john}

and

$$X \bigcup \sim X = \bigcup$$
$$\underline{P+Q}X = [X]_P \subseteq X or [X]_Q \subseteq X$$
$$= \{sam, john\}$$
$$\neq \phi$$

$$\overline{P+Q}X = \sim (\underline{P+Q}(\sim X))$$
$$= \sim (\{shyam, roger, mishra, williams, ram,$$
$$peter, hari, smith, linz, peter, lakman, fatima,$$
$$mukherjee, keny, martin, jacob, sita, pretha\})$$
$$= \{sam, john\}$$
$$\neq \bigcup$$

So X is of Type-1

$$Y \bigcup \sim Y = \bigcup$$
$$\underline{P+Q}Y = \phi$$
$$\overline{P+Q} = \{ram, albert\} \neq \bigcup$$

So Y is of Type-1

$$X \bigcup Y = \{sam, john, ram, albert\}$$
$$\overline{P+Q}(X \bigcup Y) = \sim \underline{P+Q(\sim (X \bigcup Y}$$
$$= \sim (\{hari, smith, linz, keny, martin, jacob,$$
$$sita, fatima, pretha\})$$
$$\neq \bigcup$$

$$\underline{P+Q}(X \bigcup Y) = \{sam, john, ram, albert\}$$
$$\neq \phi$$

So Union of X and Y is of Type-1

The following example shows both X and Y is of Type-1 but its union is of Type-3.

$$X = \{sam, john, sita\}$$
$$X \bigcup \sim X = \bigcup$$
$$\underline{P+Q}X = \{sam, john\} \neq \phi$$
$$\overline{P+Q}X = \{sam, john, sita, fatima\}$$
$$\neq \bigcup$$
$$Y = \{ram, albert, hari\}$$
$$Y \bigcup \sim Y = \bigcup$$
$$\underline{P+Q}Y = \{ram, albert\} \neq \phi$$
$$\overline{P+Q}Y = \{ram, albert, hari, smith, linz\}$$
$$\neq \bigcup$$

So both X and Y are of Type-1

Next to show its union if of type-3

$$X \bigcup Y = \{sam, john, sita, ram, albert, hari\}$$
$$\overline{P+Q}(X \bigcup Y) = \sim (\phi) = \bigcup$$
$$\underline{P+Q}(X \bigcup Y) = \{sam, john, ram, albert\}$$
$$\neq \phi$$
$$X \bigcup Y \text{ is of } type - 3$$

IV. CONCLUSION

In this paper the results of various types of topological property particularly union of two multigranular rough sets is validated with a faculty database table. These results would be highly useful for further studies in approximation of classification and rule generation. This could be even extended to pessimistic multigranular rough sets and also for intersection and complement also and to be worked out with a suitable database as a future work.

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